

WELSH JOINT EDUCATION COMMITTEE

CYD-BWYLLGOR ADDYSG CYMRU

General Certificate of Education

Tystysgrif Addysg Gyffredinol

Advanced Level/Advanced Subsidiary

Safon Uwch/Uwch Gyfrannol

MATHEMATICS C3

Pure Mathematics

Specimen Paper 2005/2006

(1½ hours)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

A calculator may be used for this paper.

A formula booklet is available and may be used.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Show that the equation

$$x^3 + 10x - 4 = 0$$

has a root α between 0 and 1.

The iterative formula

$$x_{n+1} = \frac{4 - x_n^3}{10}$$

with $x_0 = 0.3$ may be used to find α .

Calculate and record the values of x_1, x_2, x_3, x_4 . Write down the value of x_4 correct to five decimal places and show that it is the value of α correct to five decimal places.

[7]

2. Use Simpson's Rule with five ordinates to evaluate the integral

$$\int_1^2 \sqrt{1 + x^4} \, dx.$$

Show your working and give your answers correct to two decimal places.

[4]

3. Solve the inequality

$$|2x - 5| < 9.$$

[4]

4. (a) Given that

$$y^3 - x^2y^2 = x^2 + 3x + 1,$$

find $\frac{dy}{dx}$ in terms of x and y .

[4]

(b) Given that $x = t^3 + 2$, $y = t^2 + 3$,

find $\frac{dy}{dx}$ and show that

$$\frac{d^2y}{dx^2} = -\frac{2}{9t^4}.$$

[5]

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[5]

5. Showing all your working, find the values of θ between 0° and 360° satisfying

$$\cot^2 \theta = 7 - 2 \operatorname{cosec} \theta$$

[7]

6. Differentiate the following with respect to x , simplifying your answers as much as possible.

(a) $e^{2x} \sin x$

(b) $\frac{2x^2 - 4}{x^2 + 3}$

(c) $\tan(4x^2 + 3)$

[4], [3], [2]

7. (a) Find

(i) $\int_0^{\pi} e^{-4x+1} dx,$

(ii) $\int \left(\frac{1}{2x+1} + \frac{1}{(3x+7)^3} \right) dx.$

[7]

(b) Evaluate $\int_0^{\frac{\pi}{2}} \sin 2x dx.$

[3]

8. (a) Given that $y = \tan^{-1} x$, show that

$$\frac{dy}{dx} = \frac{1}{x^2 + 1}.$$

[3]

(b) Differentiate $\ln(x^2 + 1)$ with respect to x .

[2]

(c) Use the results derived in (a) and (b) to find

$$\int \frac{3+x}{1+x^2} dx.$$

[4]

9. Given that $f(x) = e^x$, sketch, on the same diagram, the graphs of $y = f(x)$ and $y = 2f(x) + 3$. Label any points of intersection of the graphs with the y -axis. Indicate the behaviour of the graphs for large positive and negative values of x . [5]

10. (a) The function f has domain $x \geq 2$ and is defined by

$$f(x) = \ln(2x - 3) + 4.$$

- (i) Find an expression for $f^{-1}(x)$.
(ii) State the domain and range of $f^{-1}(x)$. [6]

- (b) The functions g and h are defined for all x by

$$g(x) = x^2 + 3,$$

$$h(x) = 2x + 2.$$

Solve the equation

$$gh(x) = 2hg(x) + 15. \quad [5]$$