

494/01

MATHEMATICS P4

Pure Mathematics

A.M. FRIDAY, 14 June 2002

(1½ hours)

INSTRUCTIONS TO CANDIDATES

Answer all questions.

INFORMATION FOR CANDIDATES

Graphical calculators may be used for this paper.

The booklet 'Information for the use of candidates in Mathematics' is available and may be used.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Obtain an expression for the sum of the first n terms of the sequence whose r th term is given by

$$T_r = r(r-1).$$

Give your answer in the form $an^3 + bn$, where a, b are constants whose values are to be found. [5]

2. The function f is defined by

$$\begin{aligned} f(x) &= ax^3 + bx^2, & \text{for } x < 1, \\ f(x) &= x^2 + x, & \text{for } x \geq 1. \end{aligned}$$

Both f and its derivative are continuous for all values of x . Find the values of the constants a and b . [6]

3. Use mathematical induction to prove that

$$\frac{d^n}{dx^n} \left(\frac{1}{x} \right) = (-1)^n \frac{n!}{x^{n+1}}$$

for all positive integer values of n .

[6]

4. Find the general solution, in radians, of the equation

$$\cos 2\theta + \cos 4\theta + \cos 6\theta = 0.$$

[7]

5. The straight line joining the points $P(2p, p^2)$ and $Q(2q, q^2)$ passes through the point $(0, 1)$.

(a) (i) Show that $pq = -1$.

(ii) Deduce that the coordinates of R , the mid-point of PQ , are

$$\left(p - \frac{1}{p}, \frac{1}{2} \left(p^2 + \frac{1}{p^2} \right) \right).$$

[6]

- (b) Find the Cartesian equation of the locus of R as p varies, giving your answer in the form $y = f(x)$. [4]

6. A hyperbola has equation

$$4x^2 - y^2 - 8x - 4y - 4 = 0.$$

Find the coordinates of

- (a) the centre of the hyperbola,

[4]

- (b) the foci of the hyperbola.

[7]

7. The function f is defined on the domain $(-\infty, \infty)$ by

$$f(x) = \frac{x}{1+x^2}.$$

- (a) (i) State, with a reason, whether f is even or odd.

- (ii) Show that $-\frac{1}{2} \leq f(x) \leq \frac{1}{2}$ for all x .

- (iii) Sketch the graph of f and state the equation of the asymptote.

[9]

- (b) (i) The interval $[2, 3]$ is denoted by A . Find $f(A)$,

- (ii) The interval $\left[\frac{1}{4}, \frac{1}{2}\right]$ is denoted by B . Find $f^{-1}(B)$.

[7]

8. The roots of the cubic equation

$$x^3 - 3x^2 + 2x + 4 = 0$$

are denoted by α, β, γ . Find the cubic equation whose roots are $\frac{\beta\gamma}{\alpha}, \frac{\gamma\alpha}{\beta}, \frac{\alpha\beta}{\gamma}$.

[14]