WELSH JOINT EDUCATION COMMITTEE
General Certificate of Education
Advanced Subsidiary/Advanced

WJEC
CBAC

CYD-BWYLLGOR ADDYSG CYMRU
Tystysgrif Addysg Gyffredinol
Uwch Gyfrannol/Uwch

494/01

MATHEMATICS P4

Pure Mathematics

A.M. TUESDAY, 19 June 2001

($1\frac{1}{2}$ hours)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Only a 'scientific' calculator may be used for this paper.

INFORMATION FOR CANDIDATES

The booklet 'Information for the use of candidates in Mathematics' is available and may be used.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. For each of the following functions state, with a reason, whether it is even, odd or neither even nor odd.

   (a) $x^2 \sec x$, [2]

   (b) $1 + \sin x$, [2]

   (c) $x - \dfrac{1}{x}$. [2]

2. Use mathematical induction to prove that $4^n + 2$ is divisible by 3 for all positive integer values of n. [5]

3. Show that

$$\frac{1}{1 \times 3} + \frac{1}{2 \times 4} + \ldots + \frac{1}{n(n+2)} = \frac{3}{4} - \frac{(2n+3)}{2(n+1)(n+2)}.$$

[7]

4. Given that

$$2\cos\theta + \tan\frac{1}{2}\theta = 1,$$

use the substitution $t = \tan\frac{1}{2}\theta$ to show that

$$t^3 - 3t^2 + t + 1 = 0.$$

By first solving this equation, find all the values of θ between 0° and 360° satisfying the above equation in θ. [9]

5. The function f is defined on the domain $(0,\infty)$ by

$$f(x) = \frac{x^2 - 1}{x^2 + 1}.$$

   (a) Show that f is monotonic. [4]

   (b) State the range of f. [2]

   (c) Evaluate $f^{-1}(A)$ where A is the interval $\left(\frac{1}{2}, 1\right)$. [5]

6. A parabola has equation $y^2 = 8x$.

(a) Write down the coordinates of the focus and the equation of the directrix of the parabola. [2]

(b) Find the equation of the tangent to the parabola at the point $(2t^2, 4t)$. [3]

(c) A line which passes through the point (2,0) cuts the parabola at the points $R(2r^2, 4r)$ and $S(2s^2, 4s)$.

(i) Show that $rs = -1$.

(ii) Show that the point of intersection of the tangents to the parabola at the points R and S lies on the directrix. [8]

7. The curve C has equation

$$y = \frac{1 - x + x^2}{1 + x + x^2}.$$

(a) Show that $\frac{1}{3} \leqslant y \leqslant 3$ for all real values of x. [5]

(b) Find the coordinates of the stationary points on C. [4]

(c) Sketch C, stating the equation of the asymptote. [3]

8. The roots of the cubic equation

$$Ax^3 + Bx^2 + Cx + D = 0$$

are in geometric progression. Show that

$$AC^3 = B^3D.$$

Given that the equation

$$8x^3 - 42x^2 + 63x - 27 = 0$$

satisfies this condition, find its three roots. [12]