

1. (a) Differentiate $x^{\ln x}$ with respect to x . [4]
 (b) Use the trapezium rule with 5 ordinates to find an approximate value for

$$\int_1^2 x^{\ln x} dx.$$

Give your answer correct to three decimal places. [4]

2. (a) Find all values of x between 0° and 360° satisfying
 $3 \cos x + \sin x = -1$. [5]
 (b) Find the general solution of the equation

$$\sin 2x + \sin 4x = \cos 2x + \cos 4x. \quad [5]$$

3. Given that the cubic equation

$$x^3 - 7x + q = 0$$

has roots α , 2α and β , find the possible values of q . [6]

4. Find the expansion of

$$(1 + 3x)^{\frac{3}{2}}$$

in ascending powers of x as far as the term in x^2 . For which range of x is the expansion valid? [3]

5. (a) Simplify $\frac{1-i}{-3-i}$. [2]

- (b) Find the modulus and argument of the complex number $-5 + 12i$. [2]

6. The matrices A and C are given by

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 3 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}.$$

Find the matrix B satisfying

$$BA = C. \quad [6]$$

7. The variables x, y, z satisfy the equations

$$x + y + z = 3$$

$$x + 2y + 3z = 6$$

$$x + 3y + (k - 4)z = k.$$

- (a) Use reduction to echelon form to find the value of k for which the equations do not have a unique solution. [3]
- (b) For this value of k , find the general solution of the equations and interpret your result geometrically. [3]
8. The equation of the curve C is $y^2 = 8x$. The point $P(2t^2, 4t)$ lies on C . The line through the point $(2, 0)$ perpendicular to the tangent to C at P intersects this tangent at the point Q .
- (a) Find the coordinates of Q . [6]
- (b) Given that R is the mid-point of PQ , find the equation of the locus of R in Cartesian form. [3]
9. The function f is defined by

$$\begin{aligned} f(x) &= ax^3, & \text{for } -\infty < x < 1, \\ f(x) &= b - 4x + x^2, & \text{for } 1 \leq x < \infty, \end{aligned}$$

where a and b are constants.

The function f and its derivative are continuous for all values of x . Find the values of a and b . [3]