1. Use Simpson's rule with 5 ordinates and an interval of $\frac{\pi}{12}$ to find an approximate value for

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{\sin x}\, \mathrm{d}x$$

Give your answer correct to five decimal places. [4]

2. Differentiate the following with respect to x.

(*a*) $\displaystyle\int_0^x \frac{1}{\sqrt{t^3+1}}\,\mathrm{d}t.$ [1]

(*b*) $x^{\cos x}$. [3]

3. Given that α, β, γ are the roots of the equation

$$x^3 + x^2 + 4x - 5 = 0,$$

find the cubic equation whose roots are $\beta\gamma$, $\gamma\alpha$ and $\alpha\beta$. [4]

4. Given that

$$z = \frac{1+\mathrm{i}}{1-2\mathrm{i}},$$

find

(*a*) z in the form $a + \mathrm{i}b$, [2]

(*b*) the modulus and argument of z. [2]

5. (*a*) Show that the first three terms in the expansion in ascending powers of x of

$$(1+9x)^{\frac{1}{3}}$$

are the same as the first three terms in the expansion in ascending powers of x of

$$\frac{1+6x}{1+3x}.$$

For what values of x are both these expansions valid? [5]

(*b*) Use $(1+9x)^{\frac{1}{3}} \approx \frac{1+6x}{1+3x}$ with $x = \frac{1}{64}$ to obtain an approximation to $(73)^{\frac{1}{3}}$ as a rational fraction in its lowest terms. [2]

?

6. The unknowns x, y, z satisfy the equations

$$\begin{aligned} x + \quad y + (\lambda + 1)z &= 0, \\ x + (\lambda - 2)y + \quad 2z &= 0, \\ x - \quad y + \quad z &= \lambda - 2, \end{aligned}$$

where λ is a constant. Use reduction to echelon form to solve these equations when

(*a*) $\lambda = 1$, [3]

(*b*) $\lambda = 2$. [3]

In each case give a geometrical interpretation of your result. [2]

7. The point $P(2\cos\theta, 3\sin\theta)$ lies on the ellipse

$$\frac{x^2}{4} + \frac{y^2}{9} = 1.$$

(*a*) Find the equation of the tangent to the ellipse at the point $P(2\cos\theta, 3\sin\theta)$, where $\theta \neq 0$. [3]

(*b*) Given that the tangent in (*a*) passes through the point $(2, -6)$, show that

$$\cos\theta - 2\sin\theta = 1.$$ [1]

(*c*) Solve the equation in (*b*) for $0° \leqslant \theta \leqslant 360°$ and deduce the coordinates of P. [4]

8. Three matrices **D**, **E**, **F** are such that

$$\mathbf{EF} = \mathbf{FD}$$

and $\mathbf{F}^{-1}$ exists. Express **E** in terms of **F**, $\mathbf{F}^{-1}$ and **D**. [1]

(*a*) Show that $\mathbf{E}^9 = \mathbf{FD}^9\mathbf{F}^{-1}$. [2]

(*b*) Given that

$$\mathbf{F} = \begin{pmatrix} 1 & 1 & 1 \\ 4 & 5 & 3 \\ 1 & 1 & 1 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

find $\mathbf{E}^9$. [6]

9. The function f with domain $[0, 2]$ is defined by

$$\begin{aligned} f(x) &= x^2, && \text{for } 0 \leqslant x < 1, \\ f(x) &= 2 - x, && \text{for } 1 \leqslant x \leqslant 2. \end{aligned}$$

(*a*) Sketch the graph of f. [2]

(*b*) The even function g has domain $[-2, 2]$ and

$$g(x) = f(x) \quad \text{for } 0 \leqslant x \leqslant 2.$$