

MATHEMATICS (MODULAR) P3

Mathematical Methods 3

A.M. FRIDAY, 21 June 1996

(1½ hours)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

An electronic calculator will be required.

The booklet "Information for the use of candidates in Mathematics" is available and may be used.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Express $\cos(2x + 2h) - \cos 2x$ as a product of two sines. Hence find the derivative of $\cos 2x$ from first principles. [3]

2. The roots of the cubic equation

$$x^3 - 6x^2 + 12x - 17 = 0$$

are denoted by α , β , and γ . Find the cubic equation having roots 2α , 2β and 2γ . [2]

3. (a) Use the Trapezium Rule with 5 ordinates and an interval of 0.25 to find an approximate value for

$$\int_0^1 \frac{dx}{(1+x^2)^{\frac{3}{2}}}.$$

Give your answer correct to four decimal places. [4]

- (b) Use the substitution $x = \tan \theta$ to find the exact value of

$$\int_0^1 \frac{dx}{(1+x^2)^{\frac{3}{2}}}. \quad [3]$$

4. The function f is defined by

$$\begin{aligned} f(x) &= 4 - x^2, & \text{for } 0 \leq x \leq 1, \\ f(x) &= 2x + 1, & \text{for } 1 < x \leq 2, \\ f(x) &= 4 - (x-2)^2, & \text{for } 2 < x \leq 3. \end{aligned}$$

- (a) Sketch $f(x)$ for $0 \leq x \leq 3$. [3]

- (b) Evaluate $\int_0^3 f(x) dx$. [4]

5. Expand $\sqrt{\frac{1+4x}{1-2x}}$ as a series of ascending powers of x up to and including the term in x^2 . [3]

Substitute $x = \frac{1}{100}$ in the series and hence find an approximation for $\sqrt{13}$. [3]

6. Given that $z = \frac{3+4i}{5-12i}$, find the modulus and argument of z . [4]

7. Show that the equation

$$8 \cos^2 x - 3 \sin x \cos x - \sin^2 x = 1.5$$

can be written in the form

$$A \cos 2x + B \sin 2x = C,$$

giving the values of A , B and C . [2]

By writing this second equation in the form

$$R \cos (2x + \alpha) = S,$$

find the values of x between 0° and 360° inclusive satisfying this equation. [6]

8. Given that the matrix $A = \begin{pmatrix} 5 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 5 & 2 \end{pmatrix}$ and that the determinant of $A = 20$, find A^{-1} . [4]

9. Using reduction to echelon form or otherwise, find the value of k for which the equations

$$\begin{aligned} x + 2y + 3z &= 4 \\ 2x + 3y + z &= 1 \\ 7x + 11y + kz &= k + 1 \end{aligned}$$

do not have a unique solution. For this value of k , find the general solution. [4]

10. The points $S\left(s, \frac{1}{s}\right)$ and $T\left(t, \frac{1}{t}\right)$ lie on the curve $xy = 1$ and the line ST passes through the point $(1, 2)$.

(a) Show that $s + t = 1 + 2st$. [3]

(b) The tangents to the curve at S and T meet at the point P . Show that the locus of P is given by $y = 2 - 2x$. [7]