Thursday 15 May 2008.

1. Points A, B, C, D have coordinates

(-7, 4) (3, -1) (6, 1) (K, -15)

a. find gradient of AB.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-1-4}{3-(-7)} = \frac{-5}{10}.$$

b. find equation of AB

$: y - y_1 = m(x - x_1)$

$y - 4 = -\frac{1}{2}(x - (-7))$

$2y - 4 = -x - 7$

$2y = -x - 3$

$2y = x - 3$

c. find length of AB

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{(3-(-7))^2 + (-1-4)^2}$

$AB = \sqrt{100 + 25}$

$AB = \sqrt{125}$

d. Point E is mid point of AB
find coordinates of E

$\bar{x} = \frac{x_1 + x_2}{2}$ $\bar{y} = \frac{y_1 + y_2}{2}$

$x = \frac{-7+3}{2}$ $y = \frac{4+(-1)}{2}$

$E = \left(-2, \frac{3}{2}\right) = E\left(-2, 1.5\right)$

e. $\overrightarrow{CD}$ is perpendicular to AB
find value for constant k

$m_2 = \frac{-1}{\text{gradient of } AB (m_1)}$

$m_2 = \frac{-1}{-\frac{1}{2}} = 2$

$2 = \frac{-15 - 1}{K - 6}$

$2 = \frac{-16}{K-6}$ $2 = \frac{-16}{-2-6}$

$2 = \frac{16}{8}$

$K = -2$.

2.

a. simplify

$$\sqrt{75} - \frac{9}{\sqrt{3}} + (\sqrt{6} \times \sqrt{2})$$

$$(3\sqrt{2} \times \sqrt{2})$$

$$\sqrt{75} - \frac{9}{\sqrt{3}} + 6$$

$$15\sqrt{3} - 3\sqrt{3} + 6$$

$$12\sqrt{3} + 6$$

b. simplify.

$$\frac{5\sqrt{5} - 2}{4 + \sqrt{5}}$$

$$\frac{5\sqrt{5} - 2}{4 + \sqrt{5}} \times \frac{4 - \sqrt{5}}{4 - \sqrt{5}}$$

$$= \frac{20\sqrt{5} - 25 - 8 - 2\sqrt{5}}{16 - 4\sqrt{5} + 4\sqrt{5} - 5}$$

$$= \frac{18\sqrt{5}}{11}$$

3 point P lies on curve c with equation:

$$y = 3x^2 - 8x + 7$$

x coordinate: $P = 2$.

$P = (2, y)$

find equation of normal to c at P.

$$\frac{dy}{dx} = 6x - 8$$

gradient when $x = 2$.

$$= 6(2) - 8$$
$$= 12 - 8$$
$$= 4.$$

at P $x = 2$.

$$y = 3x^2 - 8x + 7.$$
$$y = 3(2)^2 - 8(2) + 7$$
$$y = 12 - 16 + 7$$
$$y = 3$$

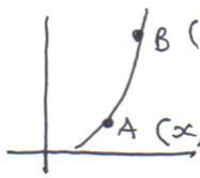
$$y - y_1 = m(x - x_1)$$
$$y - 3 = 4(x - 2)$$
$$y - 3 = 4x - 8$$

$$y = 4x - 8 + 3$$
$$y = 4x - 5$$

4.

a. given that $y = 5x^2 + 3x - 4$

find $\frac{dy}{dx}$ from first principles



$$\frac{dy}{dx} \lim_{h \to 0} \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{dy}{dx} \quad \lim_{h \to 0} \quad \frac{5(x+h)^2 + 3(x+h) - 4 - (5x^2 + 3x - 4)}{x + h - x}$$

$$\frac{dy}{dx} \quad \lim_{h \to 0} \quad \frac{\cancel{5x^2} + 5xh + 5h^2 + \cancel{3x} + 3h \cancel{-4} - \cancel{5x^2} \cancel{+4}}{\cancel{x} + h - \cancel{x}}$$

$$\frac{dy}{dx} \quad \lim_{h \to 0} \quad \frac{5xh + 5h^2 + 3h}{h}$$

$$\frac{dy}{dx} \quad \lim_{h \to 0} \quad \frac{\cancel{h}(5x + 5h + 3)}{\cancel{h}}$$

$$\frac{dy}{dx} = 5x + 8$$

b. given that $y = \frac{8}{x} + 3\sqrt{x}$

find $\frac{dy}{dx}$ when $x = 4$

$\frac{dy}{dx} = 8x^{-1} + 3x^{\frac{1}{2}}$

$= 8(4)^{-1} + 3(4)^{\frac{1}{2}}$

$2 + 6$

$\frac{dy}{dx} = 8.$

5.

a. Express $x^2 + 6x - 4$ in form $(x+a)^2 + b$.

$(x+3)^2 - 3^2 - 4$

$(x+3)^2 - 9 - 4$

$(x+3)^2 - 13.$

b. find least value of $2x^2 + 12x - 8$ using above.

$2(x^2 + 12x - 8)$

$2((x+3)^2 - 3^2 - 4)$

$2(x+3)^2 - 26$

Least value $= -26$

6. use binomial expansion theorem to expand $(a+b)^3$
$(5+2x)^3$

$(a+b)^3 = a^3 + {}^3c_1 a^2 b + {}^{\frac{3\times2}{1\times2}}c\, ab^2 + b^3$

$(a+b)^3 = (5)^3 + 3(5)^2\, 2x + 3(5)\, 2x^2 + 2x^3$

$(a+b)^3 = 125 + 75(2x) + 15(2x)^2 + 2x^3$

$(a+b)^3 = 125 + 150x + 30x^2 + 2x^3.$

7. polynomial $4x^3 + px^2 - 11x + q$ $x-2$ is factor

a. show $p=-4$ and $q=6$

$f(-1) = 4(-1)^3 + p(-1)^2 - 11(-1) + q$

$4(-1)^3 + p(-1)^2 - 11(-1) + q = 9$

$-4 + p(-1)^2 - 11 + q = 9$

$-4 + -4(-1)^2 - 11 + 6 = 9.$

b. factorise $4x^3 - 4x^2 - 11x + 6.$

$f(1) = 4(1)^3 - 4(1)^2 - 11(1) + 6$

$f(2) = 4(2)^3 - 4(2)^2 - 11(2) + 6 = 0$

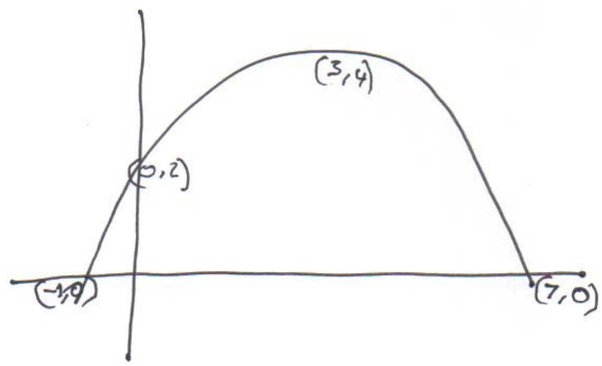
$x-2 = 0.$

$x-2 =$ factor.

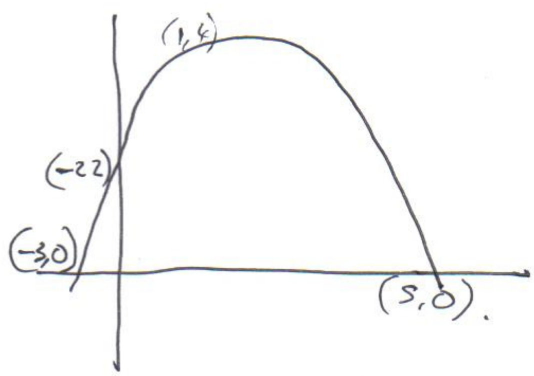
8.

graph of

$y = f(x)$



a. sketch $y = f(x+2)$



b. Sketch $y = f(x+3)$