

973/01

MATHEMATICS C1

Pure Mathematics

A.M. MONDAY, 23 May 2005

(1½ hours)

# NEW SPECIFICATION

## ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet.

## INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Calculators are **not** allowed for this paper.

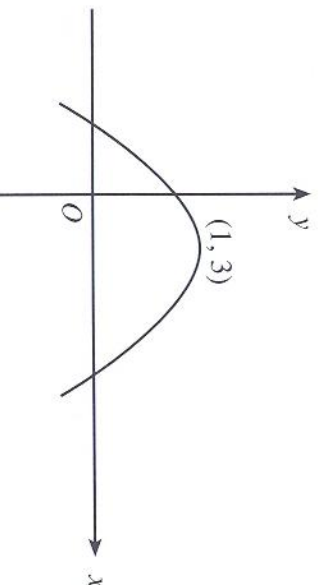
## INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The points  $A, B, C, D$  have coordinates  $(1, 7), (5, -1), (8, 3), (6, 7)$  respectively.
- (a) Show that  $AB$  and  $CD$  are parallel. [3]
  - (b) Find the equation of  $AB$ . [2]
  - (c) The line  $L$  passes through the point  $D$  and is perpendicular to  $AB$ . Show that  $L$  has equation  
$$x - 2y + 8 = 0.$$
 [3]
  - (d) The lines  $L$  and  $AB$  intersect at the point  $E$ . Find the coordinates of  $E$ . [2]
  - (e) Calculate the length of  $EF$ , where  $F$  is the mid-point of  $AB$ . [4]
2. Simplify each of the following, expressing your answers in surd form:
- (a)  $\sqrt{45} + \sqrt{80} - \sqrt{125}$ ; [3]
  - (b)  $\frac{6 + \sqrt{2}}{2 + \sqrt{2}}$ . [4]
3. (a) Given that  $x - 1$  is a factor of  $3x^3 + 5x^2 + ax - 4$ , show that  $a = -4$ . [2]  
(b) Solve the equation  $3x^3 + 5x^2 - 4x - 4 = 0$ . [4]  
(c) Calculate the remainder when  $3x^3 + 5x^2 - 4x - 4$  is divided by  $x + 1$ . [2]
4. Write down and simplify the first four terms in the binomial expansion of  $(1 + 2x)^6$ . [4]
5. Given  $y = x^2 - 7x + 2$ , find  $\frac{dy}{dx}$  from first principles. [5]
6. The curve  $C$  has equation  
$$y = 16\sqrt{x} + \frac{32}{x} + 2.$$
- (a) Find the value of  $\frac{dy}{dx}$  when  $x = 4$ . [3]
  - (b) Find the equation of the normal to  $C$  at the point where  $x = 4$ . [3]

7. The curve  $C$  has equation  $y = x^3 - 3x^2$ .
- Find the coordinates of the stationary points of  $C$  and determine the nature of each of these points. [7]
  - Sketch  $C$ . [3]
  - Find the range of values of  $k$  for which there are three real and distinct solutions of the equation  $x^3 - 3x^2 = k$ . [2]
8. (a) Express the quadratic expression  $x^2 - 6x + 16$  in the form  $(x - a)^2 + b$ , where the values of the constants  $a$  and  $b$  are to be determined. Deduce the least value of  $x^2 - 6x + 16$ . [3]
- (b) Solve the inequality
- $$(x + 1)^2 \leq 4x + 9. \quad [4]$$
9. The straight line  $y = 2x + c$  is a tangent to the curve  $y = x^2 + 6x + 7$ .
- Determine the value of the constant  $c$ . [4]
  - Find the coordinates of the point of contact of the tangent and the curve. [2]
10. The diagram shows the graph of  $y = f(x)$ . The graph has a maximum point at  $(1, 3)$ .



Sketch the following graphs, using a separate set of axes for each graph and indicating the coordinates of the stationary point in each case.

- (a)  $y = 4f(x)$  [2], [21], [2]  
 (b)  $y = f(x - 2)$  [2], [21], [2]  
 (c)  $y = f\left(\frac{x}{2}\right)$  [2], [21], [2]