

Kimberly Northam

Past paper, May 2008

- 1) $A = (-7, 4)$
 $B = (3, -1)$
 $C = (6, 1)$
 $D = (k, -15)$

Find the gradient of AB

$$\frac{4 - (-1)}{-7 - 3} = -\frac{5}{10} = -\frac{1}{2}$$

Equation of AB

$$y - y_1 = m(x - x_1) \quad A(-7, 4)$$

$$y - 4 = -\frac{1}{2}(x - (-7))$$

$$y - 4 = -\frac{1}{2}(x + 7)$$

$$\times 2 \quad 2y - 8 = -(x + 7)$$

$$2y - 8 = -x - 7$$

$$2y - 8 + 7 + x = 0$$

$$2y + x - 1 = 0$$

Length of AB

A(-7, 4) B(3, -1)

$$\sqrt{(4 - (-1))^2 + (-7 - 3)^2}$$
$$\sqrt{(5)^2 + (-10)^2}$$
$$\sqrt{25 + 100}$$
$$\sqrt{125}$$

Can you spot Kimberly's error?

E is the midpoint of AB

$$\frac{3 + (-7)}{2} = \frac{-4}{2} = -2$$
$$\frac{-1 + 4}{2} = \frac{3}{2} = 1\frac{1}{2}$$

$$E = (-2, 1\frac{1}{2})$$

What about Part (e)

Kimberly Northam

Past paper, may 2008

2) Simplify
$$\frac{\sqrt{75} - \frac{9}{\sqrt{3}} + (\sqrt{6} \times \sqrt{2})}{\sqrt{3}}$$

$$\sqrt{75} = \frac{\sqrt{25} \times \sqrt{3}}{5\sqrt{3}} \checkmark$$

$$\frac{-\frac{9}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}}{3\sqrt{3}} = \frac{9\sqrt{3}}{3} \checkmark$$

$$\begin{aligned} &(\sqrt{6} \times \sqrt{2}) \\ &\sqrt{6 \times 2} \\ &\sqrt{12} \\ &\sqrt{4 \times 3} \\ &2\sqrt{3} \checkmark \end{aligned}$$

$$\begin{aligned} &5\sqrt{3} - 3\sqrt{3} + 2\sqrt{3} \\ &2\sqrt{3} + 2\sqrt{3} \end{aligned}$$

$$\underline{4\sqrt{3}} \checkmark$$

Nice work

4 marks out of 4

b) Simplify

$$\frac{5\sqrt{5}-2}{4+\sqrt{5}} \times \frac{4-\sqrt{5}}{4-\sqrt{5}}$$

Top

	4	$-\sqrt{5}$
$5\sqrt{5}$	$20\sqrt{5}$	$-5\sqrt{5}\sqrt{5}$
-2	-8	$+2\sqrt{5}$

$$\begin{aligned} & 20\sqrt{5} - 5\sqrt{5}\sqrt{5} - 8 + 2\sqrt{5} \\ & 20\sqrt{5} - 5 \times 5 - 8 + 2\sqrt{5} \\ & 20\sqrt{5} - 25 - 8 + 2\sqrt{5} \\ & 22\sqrt{5} - 33 \end{aligned}$$

Bottom

	4	$-\sqrt{5}$
4	16	$-4\sqrt{5}$
$+\sqrt{5}$	$+4\sqrt{5}$	$-\sqrt{5}\sqrt{5}$

$$\begin{aligned} & 16 - \sqrt{5}\sqrt{5} \\ & 16 - 5 = 11 \end{aligned}$$

$$\frac{22\sqrt{5} - 33}{11}$$

~~$$\frac{2\sqrt{5} - 3}{11}$$~~

4 marks out
of 4

Kimberly Northam

Past paper, may 2008

4a) Given that $y = 5x^2 + 3x - 4$ find $\frac{dy}{dx}$ from first principles.

FORMULA

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \left(\frac{f(x+\delta x) - f(x)}{\delta x} \right)$$

$$f(x) = 5x^2 + 3x - 4$$

$$f(x+\delta x) = 5(x+\delta x)^2 + 3(x+\delta x) - 4$$
$$f(x+\delta x) = 5(x^2 + 2x\delta x + \delta x^2) + 3(x+\delta x) - 4$$
$$f(x+\delta x) = 5x^2 + 10x\delta x + 5\delta x^2 + 3x + 3\delta x - 4$$

x^2	$x\delta x$
$x\delta x$	δx^2

$$\frac{f(x+\delta x) - f(x)}{\delta x} = \frac{(5x^2 + 10x\delta x + 5\delta x^2 + 3x + 3\delta x - 4) - (5x^2 + 3x - 4)}{\delta x}$$

$$\frac{10x\delta x + 5\delta x^2 + 3\delta x}{\delta x} = \frac{f(x+\delta x) - f(x)}{\delta x}$$

$$= 10x + 5\delta x + 3$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} (10x + 5\delta x + 3)$$

$$\frac{dy}{dx} = 10x + 3$$

Rules of Indices for Differentiation

Remember -
 $\sqrt{x} = x^{1/2}$ power $x^{1/2}$ ✓

$\frac{1}{x} = x^{-1}$ ✓

$\frac{1}{x^2} = x^{-2}$ ✓

4b) $y = \frac{8}{x} + 3\sqrt{x}$, find value of $\frac{dy}{dx}$

$x = 4$.

We can not start to differentiate until there are powers.

- ★ Write expression down
- ★ Re-write using powers
- ★ Now work out the derivative (DIFFERENTIATE)

$y = \frac{8}{x} + 3\sqrt{x}$
 $= 8x^{-1} + 3x^{1/2}$
 $\frac{dy}{dx} = -1(8)x^{-1-1} + \frac{1}{2}(3)x^{1/2-1}$
 $\frac{dy}{dx} = -8x^{-2} + \frac{3}{2}x^{-1/2}$ ✓

Roots and reciprocals

hard Bit → Change back to 1 substitute $x = 4$

$\frac{dy}{dx} = \frac{-8}{x^2} + \frac{3}{2\sqrt{x}}$ ✓
 $\frac{dy}{dx} = \frac{-8}{(4)^2} + \frac{3}{2\sqrt{4}} \rightarrow \frac{-8}{16} + \frac{3}{4}$ ✓
 $-\frac{1}{2} + \frac{3}{4} = \frac{1}{4}$ ← gradient of tangent when $x = 4$.

Kimberly Northam

Past paper, may 2008.

5) Express $x^2 + 6x - 4$ in the form of $(x+a)^2 + b$.

$$(x+3) - 4 - 3^2$$

$$(x+3)^2 - 13 \quad \checkmark$$

Find the least value of $2x^2 + 12x - 8$
and the corresponding value of x

$$2x^2 + 12x - 8$$

$$2(x^2 + 6x - 4) \quad \checkmark$$

$$2((x+3)^2 - 4 - 3^2) \quad \checkmark$$

$$2((x+3)^2 - 13)$$

$$2(x+3)^2 - 26 \quad \checkmark$$

The least value is -26 \checkmark

The least value of x is -3 .

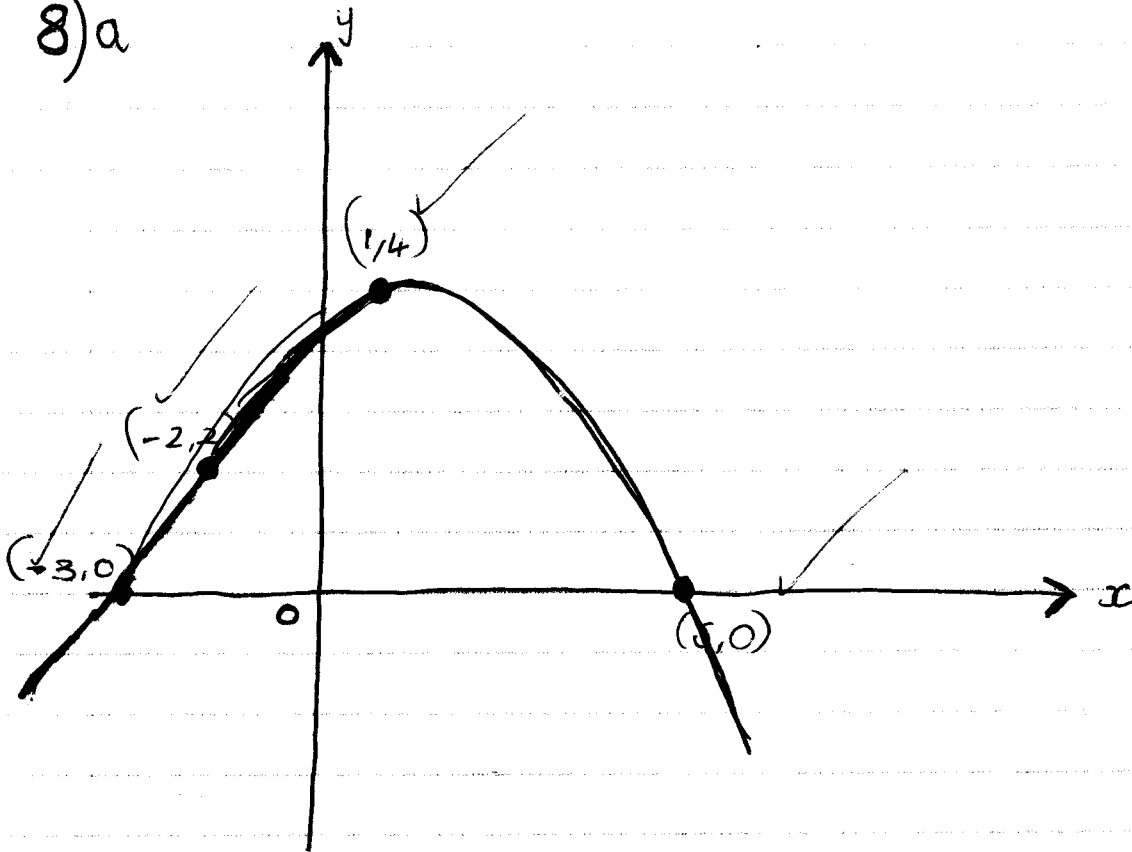
Very nice

4 marks
out of 4

Kimberly Northam

Past paper, May 2008.

8)a



Sketch the graph of $y=f(x+2)$

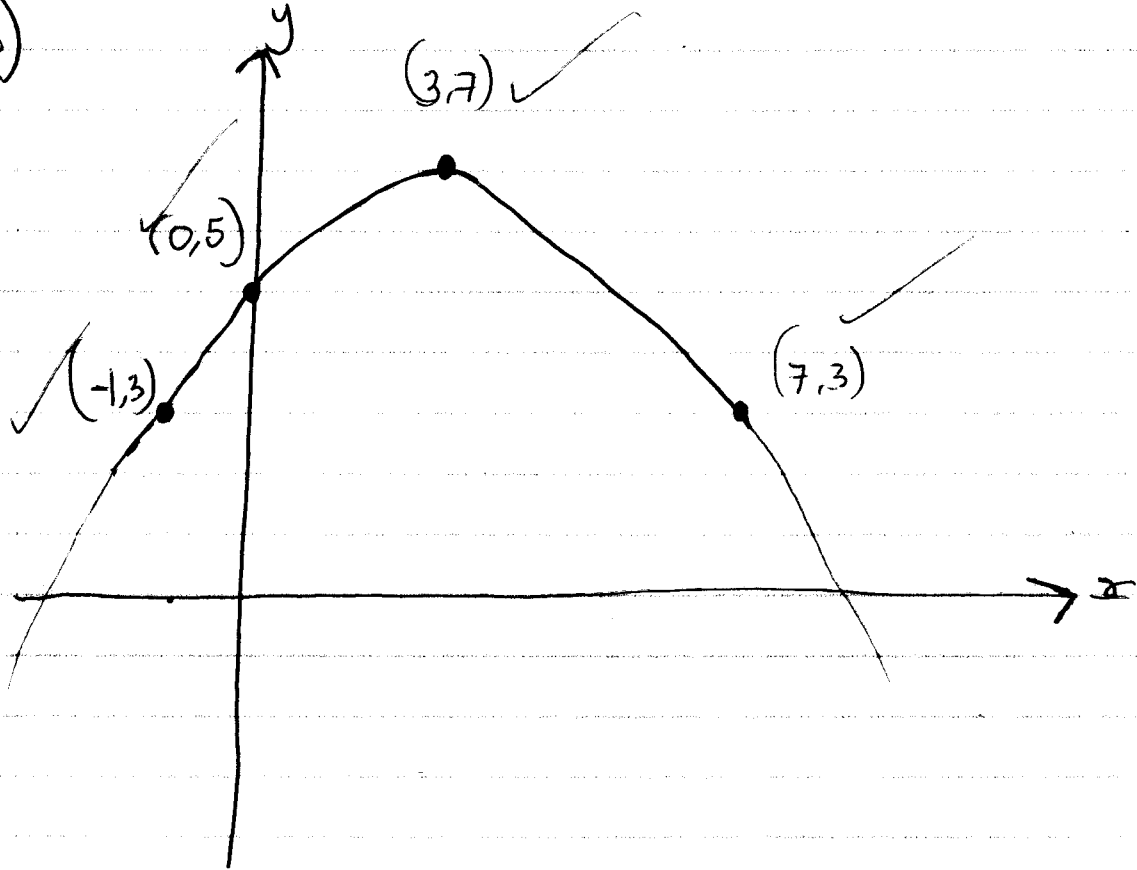
Co-ordinates are because we take away 2 from the x co-ordinate (moves left 2)

$(-1, 0)$	\rightarrow	$(-3, 0)$
$(0, 2)$	\rightarrow	$(-2, 2)$
$(3, 4)$	\rightarrow	$(1, 4)$
$(7, 0)$	\rightarrow	$(5, 0)$

Kimberly Northam

Past paper, May 2008.

8b)



Sketch the graph $y = f(x) + 3$

Co-ordinates are
 $(-1, 0) \rightarrow (-1, 3)$
 $(0, 2) \rightarrow (0, 5)$
 $(3, 4) \rightarrow (3, 7)$
 $(7, 0) \rightarrow (7, 3)$

because we
add 3 to the
y co-ordinate.
(moves up 3)

Kimberly Northam

Past paper, May 2008.

10) Solve the inequality
 $2x^2 - 3x - 9 \geq 0$

Criticals $\rightarrow +3 \cdot 6$
 $2 \times 9 = 18$

$$2x^2 - 6x + 3x - 9 \neq 0$$

$$2x(x-3) - 3(x-3)$$

$$(2x-3)(x-3)$$

$$x = \frac{3}{2}$$

$$\text{OR } x = 3$$

= 0 for criticals

error in signs here.

Can you see
what the
correct
factorisation
should be?

You have found the Criticals (with error)
Now what values of x
are the solution to the inequality?

b) TARGET

The Discriminant