

GCE AS/A level

981/01

MATHEMATICS M2 Mechanics 2

A.M. FRIDAY, 11 June 2010 $1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer all questions.

Take g as 9.8 ms $^{-2}$.

Sufficient working must be shown to demonstrate the mathematical method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question. You are reminded of the necessity for good English and orderly presentation in your answers. 1. A particle *P* moves in a straight line so that its acceleration $a \,\mathrm{ms}^{-2}$ at time *t*s, is given by

$$a = 3 - 4t$$
.

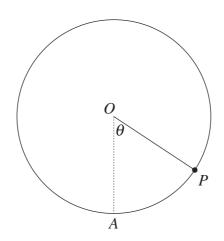
At time t = 0, the particle P passes through the point O and its velocity is -1 ms^{-1} .

- (a) Find an expression for the velocity of P at time ts. [4]
- (b) Find the values of t when P is instantaneously at rest. [2]
- (c) Find the distance between the points at which P is instantaneously at rest. [4]
- 2. At time t s, the position vector \mathbf{r} m of a particle P is given by

$$\mathbf{r} = (3t^2 + 1)\mathbf{i} + (13t - 2t^2)\mathbf{j}$$

- (a) Find the speed of P when t = 2. [4]
- (b) Calculate the value of t when the velocity of P is perpendicular to the vector $2\mathbf{i} \mathbf{j}$. [3]
- (c) Show that the acceleration of *P* is constant and find its magnitude. [3]
- (d) Find the angle between the direction of the acceleration of P and the direction of the velocity of P when t = 2. [3]
- 3. A particle *P*, of mass 3 kg, is attached to one end *A* of a light elastic string of natural length 2 m. The other end *B* of the string is attached to a point on the ceiling. The modulus of elasticity of the string is 294 N.
 - (a) The particle P is suspended in equilibrium. Calculate the extension of the string AB with A vertically below B.[3]
 - (b) The particle P is held at a distance of $1 \cdot 2$ m vertically below B and is then released. Determine the speed of P as it passes through the equilibrium position. [8]
- 4. The engine of a vehicle, of mass 1500 kg, works at a constant rate of 30 kW. The vehicle is moving up a slope inclined at an angle α to the horizontal, where $\sin \alpha = \frac{6}{49}$. The resistance to motion is a constant force of magnitude 600 N.
 - (a) Determine the acceleration of the vehicle when its speed is 8 ms^{-1} . [6]
 - (b) Find the maximum speed that can be attained by the vehicle. [4]

- 5. The point A is at the top of a vertical cliff 39.2 m above sea level. A pebble is projected from point A with speed $V \text{ ms}^{-1}$ at an angle of 30° above the horizontal. The greatest height reached by the pebble is 4.9 m above A.
 - (a) Show that V = 19.6. [4]
 - (b) Calculate the time taken for the pebble to reach the surface of the sea. [4]
 - (c) Find, correct to 3 significant figures, the speed of the pebble 3 s after projection. [5]
- 6. An athlete is cycling at a constant speed $v \,\mathrm{ms}^{-1}$ in a horizontal circle, of radius 40 m, on a track that is banked at an angle of 30° to the horizontal. The combined mass of the bicycle and the athlete is 60 kg and the coefficient of friction between the bicycle tyres and the track is $\frac{1}{4}$. Find, correct to three significant figures, the greatest possible value of *v*. [7]
- 7. The diagram shows a particle *P*, of mass 3 kg, attached by a light inextensible string of length 2.5 m to a fixed point *O*. Initially, *P* is projected from its lowest point *A* with a horizontal speed of 13 ms^{-1} so that it starts to move in a vertical circle with centre *O*.



(a) Find an expression, in terms of θ , for the speed of P when OP makes an angle θ with OA.

Find the speed of P when $\cos \theta = \frac{1}{2}$.

(b) Find an expression, in terms of θ , for the tension in the string when *OP* makes an angle θ with *OA*. [4]

[5]

(c) Determine whether or not *P* describes complete circles. [2]