WELSH JOINT EDUCATION COMMITTEE
General Certificate of Education
Advanced Subsidiary/Advanced

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# MATHEMATICS C3

## Pure Mathematics

P.M. THURSDAY, 16 June 2005

($1\frac{1}{2}$ hours)

**NEW SPECIFICATION**

### ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

### INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

### INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

**1.** Use Simpson's Rule with five ordinates to find an approximate value for

$$\int_0^1 \sqrt{1+x^5}\,\mathrm{d}x.$$

Show your working and give your answer correct to three decimal places. [4]

**2.** *(a)* Sketch the graphs of $y = x^4$ and $y = 1 - 3x$. Deduce the number of real roots of the equation

$$x^4 + 3x - 1 = 0.$$ [3]

*(b)* Show that the equation

$$x^4 + 3x - 1 = 0$$

has a root $\alpha$ between 0 and 1.

The recurrence relation

$$x_{n+1} = \frac{1 - x_n^4}{3}$$

with $x_0 = 0{\cdot}3$ can be used to find $\alpha$. Find and record the values of $x_1$, $x_2$, $x_3$, $x_4$. Write down the value of $x_4$ correct to five decimal places and prove that this value is the value of $\alpha$ correct to five decimal places. [7]

**3.** *(a)* Show, by counter-example, that the statement

$$\cot^2\theta \equiv 1 + \operatorname{cosec}^2\theta$$ [2]

is false.

*(b)* Find all values of $\theta$ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$10\sec^2\theta = 11\tan\theta + 16.$$ [6]

**4.** *(a)* A function is defined implicitly by

$$x^2 + 2xy + 3y^2 = 12.$$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $x$ and $y$. [3]

*(b)* Another function is defined parametrically by $x = 2t^4$, $y = 3t^2$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in term of $t$.

(ii) Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ in terms of $t$. [4]

**5.** *(a)* Sketch the graph of $y = |x|$ for values of $x$ from $x = -2$ to $x = 2$. [2]

*(b)* Solve the equation $|2x| + 3 = 4$. [1]

*(c)* Solve the inequality $|3x + 4| > 5$. [3]

**6.** *(a)* Differentiate each of the following with respect to $x$ and simplify your answers.

(i) $e^{2x-5}$ (ii) $x^2 \ln x$ (iii) $(3x^2 + 2)^4$ [8]

*(b)* By first writing $\tan x = \dfrac{\sin x}{\cos x}$, show that $\dfrac{d}{dx}(\tan x) = \sec^2 x$. [3]

*(c)* By first writing $y = \tan^{-1} x$ as $x = \tan y$, show that $\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1 + x^2}$. [3]

**7.** *(a)* Find (i) $\displaystyle\int \frac{1}{(3x+7)}\,dx$ (ii) $\displaystyle\int e^{3x+2}\,dx$ (iii) $\displaystyle\int \frac{3}{(5x+2)^4}\,dx$ . [6]

*(b)* Evaluate $\displaystyle\int_0^{\frac{\pi}{6}} \sin\left(4x + \frac{\pi}{6}\right)dx$, writing your answer in surd form. [4]

**8.** Given $f(x) = \ln x$, sketch on the same diagram the graphs of $y = f(x)$ and $y = 4f(x-1)$. Label the coordinates of the point of intersection of each of the graphs with the $x$-axis. Indicate the behaviour of each of the graphs for large positive and negative values of $y$. [5]

**9.** The function $f$ has domain $(2, \infty)$ and is defined by

$$f(x) = \ln(x-2) + 3.$$

Find an expression for $f^{-1}(x)$. [4]

**10.** The functions $f$ and $g$ have domains $(0, \infty)$ and $(5, \infty)$ respectively, and are defined by

$$f(x) = x^2 + 1,$$
$$g(x) = 2x - 3.$$

*(a)* Write down the ranges of $f$ and $g$. [2]

*(b)* Give the reason why $gf(1)$ cannot be formed. [1]

*(c)* Solve the equation [4]

$$fg(x) = 3x^2 - 6x + 17.$$