979/01
MATHEMATICS FP3
Further Pure Mathematics
A.M. FRIDAY, 22 June 2007
(1 $1 \frac{1}{2}$ hours)

## ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.


## INSTRUCTIONS TO CANDIDATES

Answer all questions.

## INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

1. (a) Use the substitution $x=2 \sinh \theta-1$ to evaluate the integral

$$
\begin{equation*}
\int_{0}^{1} \frac{\mathrm{~d} x}{\sqrt{x^{2}+2 x+5}} \tag{8}
\end{equation*}
$$

2. The function $f$ is defined by

$$
f(x)=x^{3}+3 x^{2}+6 x-5 .
$$

(a) Show that $f$ is strictly increasing for all values of $x$. Deduce the number of real roots of the equation $f(x)=0$.
(b) (i) Show that the equation $f(x)=0$ has a root in the interval $[0,1]$.
(ii) Use the Newton-Raphson method to find the value of this root correct to four decimal places.
3.


The above diagram shows the upper half of the circle with equation $x^{2}+y^{2}=a^{2}$.
(a) Show that, on this curve,

$$
\begin{equation*}
1+\left(\frac{\mathrm{d} y}{\mathrm{~d} x}\right)^{2}=\frac{a^{2}}{y^{2}} \tag{4}
\end{equation*}
$$

(b) Hence show that the curved surface area of a sphere with radius $a$ is equal to $4 \pi a^{2}$.
4.


The diagram shows the initial line, the line $\theta=\frac{\pi}{2}$ and the curves $C_{1}, C_{2}$ with equations $C_{1}: r=\mathrm{e}^{\theta}\left(0 \leqslant \theta \leqslant \frac{\pi}{2}\right)$,
$C_{2}: r=2 \mathrm{e}^{-\theta}\left(0 \leqslant \theta \leqslant \frac{\pi}{2}\right)$.
(a) Find the polar coordinates of the point of intersection of $C_{1}$ and $C_{2}$.
(b) Find the area of the shaded region.
5. (a) Given that $a \cosh x+b \sinh x \equiv r \cosh (x+\alpha)$ where $a>b>0, r>0$, show that

$$
\alpha=\frac{1}{2} \ln \left(\frac{a+b}{a-b}\right)
$$

and find an expression for $r$ in terms of $a$ and $b$.
(b) Hence, or otherwise, solve the equation

$$
5 \cosh x+3 \sinh x=4
$$

giving your answer as a natural logarithm.
6. The function $f$ is defined by

$$
f(x)=\ln \tan \left(\frac{\pi}{4}+x\right)
$$

(a) Show that

$$
\begin{equation*}
f^{\prime}(x)=2 \sec 2 x \tag{4}
\end{equation*}
$$

(b) Find the first two non-zero terms in the Maclaurin expansion of $f$.
(c) The equation

$$
f(x)=10 x^{3}
$$

has a small positive root. Find its approximate value.
7. The integral $I_{n}$ is defined, for $n \geqslant 0$, by

$$
I_{n}=\int_{0}^{1} x^{n}(1-x)^{\frac{3}{2}} \mathrm{~d} x
$$

(a) Show that, for $n \geqslant 1$,

$$
\begin{equation*}
I_{n}=\left(\frac{2 n}{2 n+5}\right) I_{n-1} \tag{7}
\end{equation*}
$$

(b) Evaluate $I_{2}$.

