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# MATHEMATICS FP2

## Further Pure Mathematics

A.M. MONDAY, 19 June 2006

($1\frac{1}{2}$ hours)

## ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

## INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

## INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

**1.** The function $f$ is defined as follows.

$$f(x) = x \qquad \text{for } x < 0,$$

$$f(x) = \sin x \qquad \text{for } x \geqslant 0.$$

Determine whether or not

(i) the function $f$,

(ii) its derivative $f'$

is continuous when $x = 0$. [5]

**2.** Find the three cube roots of the complex number i. Give your answers in the form $x + iy$. [9]

**3.** The function $f$ is defined on the domain $(-\infty, 0) \cup (0, \infty)$ by

$$f(x) = \frac{1}{x(x^2 + 1)}.$$

*(a)* Show that $f$ is strictly decreasing over the interval $(0, \infty)$. [3]

*(b)* State, giving a reason, whether $f$ is even or odd or neither even nor odd. [2]

*(c)* State the equation of each of the asymptotes on the graph of $f$. [2]

*(d)* Sketch the graph of $f$. [2]

**4.** A hyperbola has equation

$$2x^2 - 4x - y^2 - 4y = 4.$$

*(a)* Find the coordinates of the centre of the hyperbola. [4]

*(b)* Find the coordinates of the foci and the equations of the directrices. [5]

**5.** By putting $t = \tan\left(\frac{\theta}{2}\right)$, find the general solution of the equation

$$3\cos\theta + 4\sin\theta = 3 - \tan\left(\frac{\theta}{2}\right).$$

[9]

**6.** *(a)* Use mathematical induction to prove that

$$(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta$$

where $n$ is a positive integer. [6]

*(b)* Use the result in *(a)* with $n = 5$ to show that

$$\sin 5\theta = a\sin^5\theta - b\sin^3\theta + c\sin\theta$$

where $a, b, c$ are positive integers to be found. [7]

**7.** *(a)* Express

$$\frac{x}{(x+2)(x^2+4)}$$

in partial fractions. [4]

*(b)* Hence evaluate the integral

$$\int_2^3 \frac{x}{(x+2)(x^2+4)}\,\mathrm{d}x,$$

giving your answer correct to three decimal places. [6]

**8.** The line $y = m(x - 2)$ intersects the circle $x^2 + y^2 = 1$ at the points $A$ and $B$.

*(a)* Show that the coordinates of $M$, the mid-point of $AB$, are

$$\left(\frac{2m^2}{1+m^2}, -\frac{2m}{1+m^2}\right).$$

[5]

*(b)* Find the Cartesian equation of the locus of $M$ as $m$ varies. [6]