

WELSH JOINT EDUCATION COMMITTEE CYD-BWYLLGOR ADDYSG CYMRU

General Certificate of Education

Tystysgrif Addysg Gyffredinol

Advanced Level/Advanced Subsidiary

Safon Uwch/Uwch Gyfrannol

MATHEMATICS FP2

Further Pure Mathematics

Specimen Paper 2005/2006

(1½ hours)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

A calculator may be used for this paper.

A formula booklet is available and may be used.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The function g is defined by

$$\begin{aligned} g(x) &= x + bx^2 \quad \text{for } x \leq 1, \\ g(x) &= 2 + ax^3 \quad \text{for } x > 1. \end{aligned}$$

Given that $g(x)$ and its derivative are continuous for all values of x , find the values of the constants a and b . [6]

2. Find all the values of θ in the interval $[0^\circ, 180^\circ]$ satisfying the equation

$$\sin \theta - \sin 3\theta + \sin 5\theta = 0. \quad [8]$$

3. Find the three cube roots of the complex number $3 - 2i$. Give your answers in the form $x + iy$, with x and y correct to two decimal places. [11]

4. The function f is defined on the domain $(1, \infty)$ by

$$f(x) = \frac{2x+1}{x-1}.$$

- (a) Show that f is strictly decreasing. [3]

- (b) State the range of f . [2]

- (c) Given that S denotes the interval $[3, 4]$, determine

(i) $f(S)$,

(ii) $f^{-1}(S)$. [6]

5. Given that

$$z = \cos \theta + i \sin \theta,$$

use de Moivre's Theorem to show that

$$z^n - \frac{1}{z^n} = 2i \sin n\theta.$$

Hence, by expanding $\left(z - \frac{1}{z}\right)^5$ show that

6. A parabola has equation

$$y^2 + 4y - 8x + 12 = 0.$$

- (a) Determine the coordinates of
- (i) the vertex,
 - (ii) the focus. [4]
- (b) (i) Verify that the point $P(2p^2 + 1, 4p - 2)$ lies on the parabola for all values of p .
- (ii) Find the equation of the tangent to the parabola at P .
- (iii) Hence show that the gradients of the two tangents from the origin to the parabola are

$$\frac{2}{1 \pm \sqrt{3}}. \quad [10]$$

7. (a) The function f is defined by

$$f(x) = \frac{1}{(x+1)(x^2+4)} \quad (x \neq -1).$$

- (i) Sketch the graph of f .
 - (ii) State the equations of all the asymptotes. [4]
- (b) Express $f(x)$ in partial fractions. [4]
- (c) Hence evaluate the integral

$$\int_0^1 f(x) dx,$$

giving your answer correct to three significant figures. [8]