



GCE AS/A level

978/01

MATHEMATICS FP2

Further Pure Mathematics

P.M. FRIDAY, 19 June 2009

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The functions f , g and h are defined as follows:

$$f(x) = \sin x$$

$$g(x) = |x|$$

$$h(x) = \frac{1}{x}$$

- (a) State, with a reason, which one of the above functions is not continuous. [2]

- (b) State, with a reason, whether

(i) g is even or odd,

(ii) h is even or odd. [4]

2. Using the substitution $u = \tan x$, evaluate the integral

$$\int_0^{\frac{\pi}{6}} \frac{\sec^2 x}{\sqrt{3 - \sec^2 x}} dx .$$

Explain briefly why the integral could not be evaluated if the upper limit were changed to $\frac{\pi}{3}$. [7]

3. Giving your answers in the form $r(\cos\theta + i\sin\theta)$, find the fourth roots of the complex number $-8 + 8\sqrt{3}i$. [8]

4. Find the general solution to the equation

$$\sin\theta + \sin 2\theta + \sin 3\theta = 0. [7]$$

5. The function f is defined by

$$f(x) = \frac{1}{(x+1)(x+2)(x+3)} .$$

- (a) Express $f(x)$ in partial fractions. [4]

- (b) Evaluate the integral

$$\int_0^5 f(x) dx ,$$

giving your answer in the form $\ln\left(\frac{m}{n}\right)$ where m, n are integers. [5]

6. The ellipse E has equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad .$$

(a) Show that the equation of the tangent to E at the point $(a\cos\theta, b\sin\theta)$ is

$$bx\cos\theta + aysin\theta = ab. \quad [5]$$

(b) This tangent meets the coordinate axes at P and Q , and the mid-point of PQ is R . Find the Cartesian equation of the locus of R as θ varies. [7]

7. (a) Given that

$$z = \cos\theta + i\sin\theta,$$

show that

$$z^n + z^{-n} = 2\cos n\theta. \quad [3]$$

(b) Hence solve the equation

$$z^2 - 2z + 3 - 2z^{-1} + z^{-2} = 0. \quad [7]$$

8. The function f is defined by

$$f(x) = \frac{x(x+3)}{x-1} \quad .$$

(a) Show that $f(x)$ can be written in the form

$$ax + b + \frac{c}{x-1}$$

where a, b, c are constants to be found. [3]

(b) Find the coordinates of the stationary points on the graph of f . [4]

(c) State the equation of each of the asymptotes on the graph of f and sketch the graph of f . [4]

(d) Find $f^{-1}(A)$, where A is the interval $[0, 10]$. [5]