CYD-BWYLLGOR ADDYSG CYMRU
Tystysgrif Addysg Gyffredinol Uwch Gyfrannol/Uwch

977/01

## MATHEMATICS FP1

Further Pure Mathematics
P.M. MONDAY, 12 June 2006
(1 $\frac{1}{2}$ hours)

## ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.


## INSTRUCTIONS TO CANDIDATES

Answer all questions.

## INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

1. Find an expression for the sum of the series

$$
\begin{equation*}
1.2 .6+2.3 .7+\ldots+n(n+1)(n+5) . \tag{6}
\end{equation*}
$$

Give your answer as a product of linear factors in $n$.
2. Differentiate $\frac{1}{2 x-3}$ from first principles.
3. Solve the following equation for the complex number $z$.

$$
\frac{z}{z+1}=2+3 \mathrm{i}
$$

Give your answer in its simplest form.
4. Consider the cubic equation

$$
x^{3}+p x^{2}+56 x+q=0
$$

Given that the three roots are all positive and are the first three terms of a geometric series with common ratio 2,
(a) find the three roots of the equation,
(b) find the values of $p$ and $q$.
5. The matrices $\mathbf{A}$ and $\mathbf{I}$ are given by

$$
\mathbf{A}=\left[\begin{array}{rrr}
-4 & -4 & 4 \\
-1 & 0 & 1 \\
-7 & -6 & 7
\end{array}\right] ; \quad \mathbf{I}=\left[\begin{array}{lll}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right]
$$

(a) Write down the matrix $\mathbf{A}+\lambda \mathbf{I}$.
(b) Find the values of $\lambda$ for which the matrix $\mathbf{A}+\lambda \mathbf{I}$ is singular.
6. The transformations $T_{1}$ and $T_{2}$ in the plane are defined as follows. $T_{1}$ : A translation in which the point $(x, y)$ is transformed to the point $(x-1, y+1)$.
$T_{2}$ : A reflection in the line $y=x$.
The single transformation $T$ is equivalent to $T_{1}$ followed by $T_{2}$.
(a) Find the $3 \times 3$ matrix representing $T$.
(b) Find the equation of the locus of the fixed points of $T$.
7. Use mathematical induction to show that $9^{n}-5^{n}$ is divisible by 4 for all positive integers $n$
8. Use reduction to echelon form to solve the equations

$$
\left[\begin{array}{rrr}
1 & 3 & 2  \tag{7}\\
2 & 1 & 1 \\
3 & 2 & -1
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]=\left[\begin{array}{r}
13 \\
7 \\
4
\end{array}\right] .
$$

9. The curve $C$ has equation

$$
y=x^{-x} \text { for } x>0
$$

(a) Find the coordinates of the stationary point on $C$.
(b) (i) Show that

$$
y \frac{\mathrm{~d}^{2} y}{\mathrm{~d} x^{2}}=\left(\frac{\mathrm{d} y}{\mathrm{~d} x}\right)^{2}-\frac{y^{2}}{x}
$$

(ii) Hence determine the nature of this stationary point.
10. The complex numbers $z$ and $w$ are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$
w=z^{2} .
$$

The point $P$ moves along the line $y=x-1$. Find the Cartesian equation of the locus of $Q$.

