WELSH JOINT EDUCATION COMMITTEE
General Certificate of Education
Advanced Subsidiary/Advanced

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977/01

MATHEMATICS FP1

Further Pure Mathematics

P.M. TUESDAY, 23 January 2007

($1\frac{1}{2}$ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Find an expression, in terms of n, for the sum of the series

$$1.2.3 + 2.3.5 + 3.4.7 + \dots + n(n+1)(2n+1).$$

Express your answer as a product of linear factors. [5]

2. *(a)* Find the inverse of the following matrix.

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}$$ [6]

(b) **Hence** solve the equations

$$\begin{aligned} x + 2y + z &= 1 \\ 2x + 3y + z &= 4 \\ 3x + 4y + 2z &= 4. \end{aligned}$$ [2]

3. *(a)* Showing your working, simplify the expression

$$\frac{(3+4\mathrm{i})(1+2\mathrm{i})}{1+3\mathrm{i}}$$

giving your answer in the form $x + \mathrm{i}y$. [4]

(b) Write down, in terms of $\arg(z_1)$ and $\arg(z_2)$,

(i) $\arg(z_1 z_2)$,

(ii) $\arg\left(\frac{z_1}{z_2}\right)$. [1]

(c) Use the results in *(a)* and *(b)* to show that

$$\tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1} 2 - \tan^{-1} 3 = \frac{\pi}{k}$$

where k is a positive integer whose value is to be determined. [2]

4. Use mathematical induction to show that $6^n + 4$ is divisible by 5 for all positive integers n. [7]

5. Consider the simultaneous equations

$$\begin{aligned} x + 2y - z &= 2 \\ 2x - y + z &= 3 \\ 4x - 7y + 5z &= 5. \end{aligned}$$

Given that these equations do not have a unique solution,

(a) show that the equations are consistent. [4]

(b) find the general solution to the equations. [3]

6. The function f is defined on the domain $(0, \infty)$ by

$$f(x) = x^{-\ln x}$$

(a) Find the coordinates of the stationary point on the graph of f. [5]

(b) Determine the nature of this stationary point. [2]

7. The roots of the cubic equation

$$x^3 + 2x^2 + 3x - 4 = 0$$

are denoted by α, β, γ. Find the cubic equation whose roots are $\dfrac{\beta\gamma}{\alpha}, \dfrac{\gamma\alpha}{\beta}, \dfrac{\alpha\beta}{\gamma}$. [11]

8. The transformation T in the plane consists of an anticlockwise rotation about the origin through an angle θ followed by a translation in which the point (x, y) is transformed to the point $(x + h, y + k)$.

(a) Find the 3×3 matrix corresponding to T. [4]

(b) Given that T maps the point $(0, 1)$ to $(1, 2)$ and the point $(3, 0)$ to $(4, 3)$, find the values of h, k and θ. [7]

9. The complex number z is represented by the point $P(x, y)$ in an Argand diagram.

(a) Given that

$$|z - 3| = |z + \mathrm{i}|,$$

find the Cartesian equation of the locus of P. [5]

(b) Find the coordinates of the two points lying on this locus for which $|z| = 4$. [7]