Even Functions

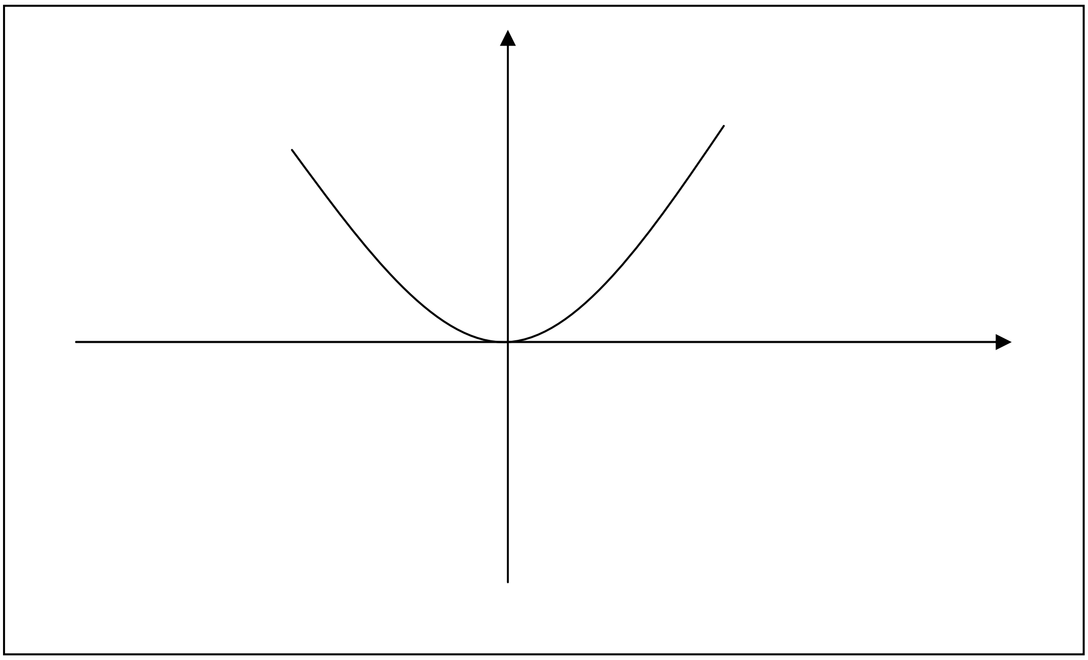
An **EVEN function** is such that

$$f(-x) = f(x)$$

for all values of x on it's domain

The graph of an EVEN FUNCTION is therefore SYMMETRIC ABOUT THE X AXIS.

$$f : x \to x^2, \quad x \in \mathbb{R}$$



Other important even functions are cosx which is a periodic Trigonometric function.
Also
Coshx which is a **HYPERBOLIC FUNCTION** (meet later in FP3).

ODD Functions

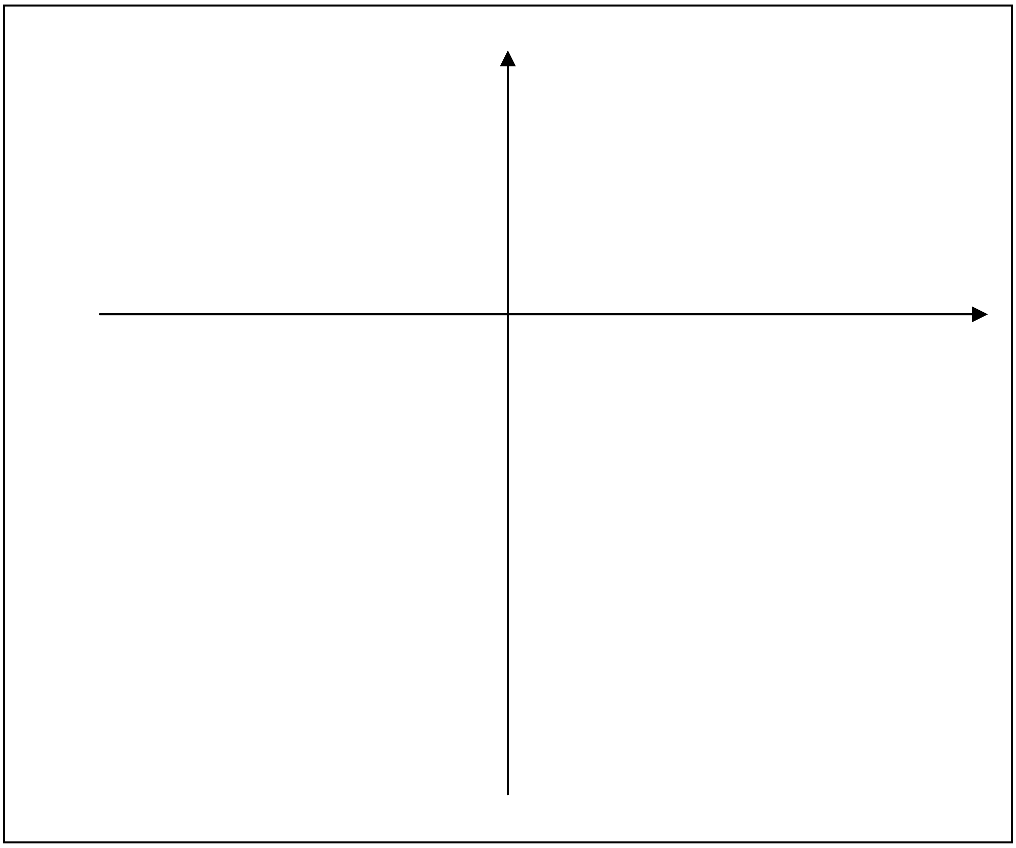
An **<u>ODD Function</u>** is such that

$$f(-x) = -f(x)$$

For all values of x in its domain.

The graph of an ODD function looks the same when ROTATED THROUGH HALF A REVOLUTION about the ORIGIN.

$$f : x \rightarrow x^3, \quad x \in \mathbb{R}$$



PROPERTIES

- If f(x) and g(x) are EVEN functions then so is af(x)+bg(x) where a and b are constants.
- If f(x) and g(x) are EVEN functions then so is f(x)g(x).
- Likewise the product of two ODD functions is an EVEN function.
- If f(x) and g(x) are ODD functions then so is af(x)+bg(x) where a and b are constants.
- The product of an ODD function with and EVEN function is an ODD function.

MONOTONIC FUNCTIONS

A monotonic function is a function that is either always increasing or always decreasing

The gradient is therefore of one sign only for all values of x in the domain

$$f'(x) > 0$$

or

$$f'(x) < 0$$

Monotonic functions are one to one functions and so have an inverse.

So if a function is monotonic we can make

$f'(x) > 0$

or

$f'(x) < 0$

f(x) is a 1 to 1 function.

f(x) has an inverse

$f^{-1}(x)$ exists.

EXAMPLE

Prove that $f(x) = \dfrac{x-1}{x-2}$ is monotonic on $(2, \infty)$

To do this we must differentiate and investigate the sign of the derivative for all values of x on the domain.

You will need to make notes from FURTHER PURE MATHEMATICS
Bostock, Chandler and Rourke (the orange book)

Page 177 to 190
This includes an EXERCISE 4a on page 188 to 190.

You should work through this exercise ignoring questions that have not yet been covered (mainly Integration)