ROW REDUCTION
(REDUCING A MATRIX TO ECHELON FORM)

A matrix in ECHELON form is of the form:

$$\begin{pmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{pmatrix} \text{ or } \begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix}$$ The zero entries above or below the lead diagonal.

Consider traditional methods of solving simultaneous equations. It involves adding or subtracting multiples of one equation to another in order to eliminate variables.The same principles are involved in solving simultaneous equations by reducing to echelon form.

SOLVE BY REDUCTION TO ECHELON FORM

$$x+3y+z=6$$
$$2x+4y+3z=16$$
$$x-y-2z=-16$$

$$\begin{pmatrix} 1 & 3 & 1 & 6 & \vdots & 11 \\ 2 & 4 & 3 & 16 & \vdots & 25 \\ 1 & -1 & -2 & -6 & \vdots & -8 \end{pmatrix}$$

The 4th column is made up of the constant terms in each equation.

This is called the AUGMENTED MATRIX of the Simultaneous equations. It is the starting point of the process.

The final column is put in as a ROW SUM CHECK. It consists of the total of the coefficients of each row.

R_2-2R_1

R_3-R_1

$$\begin{pmatrix} 1 & 3 & 1 & 6 & \vdots & 11 \\ 0 & -2 & 1 & 4 & \vdots & 3 \\ 0 & -4 & -3 & -12 & \vdots & -19 \end{pmatrix}$$

Here we must "pivot" about Row 1 ie. add multiples of Row 1 to Row 2 and Row 3 to make the first entry in each row become zero.

R_3-2R_2

$$\begin{pmatrix} 1 & 3 & 1 & 6 & \vdots & 11 \\ 0 & -2 & 1 & 4 & \vdots & 3 \\ 0 & 0 & -5 & -20 & \vdots & -25 \end{pmatrix}$$

Next "pivot" about Row 2 to make the second entry in Row 3 become zero.

The row sum checks still hold. They are merely checks and can be missed out.

THE MATRIX IS NOW IN ECHELON FORM.
Now rewrite the equations with the appropriate variables.

$$x + 3y + z = 6 \longrightarrow 1$$
$$-2y + z = 4 \longrightarrow 2$$
$$-5z = -20 \longrightarrow 3$$

from equation 3 we can directly find z since the other two variables have been eliminated.

$$-5z = -20$$
$$z = 4$$

Now substituting this into equation 2

$$-2y + z = 4$$
$$-2y + 4 = 4$$
$$-2y = 0$$
$$y = 0$$

And finally substituting into equation 1

$$x + 3y + z = 6$$
$$x + 0 + 4 = 6$$
$$x = -4$$

So our solution is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$

NOTE: Since matrix rows represent equations we may also

- Multiply or divide any row by any number
- Interchange Rows.

ANOTHER EXAMPLE

Solve by reducing to ECHELON FORM.

$$\begin{aligned} 2x - 3y + 4z &= 11 \\ x + y + z &= 2 \\ 3x - y - 2z &= 0 \end{aligned}$$

Try this for yourself

The solutions should be:

$$x = \frac{31}{33}, \quad y = \frac{-23}{33}, \quad z = \frac{58}{33}$$