May 2008 past paper Craig Mansfield

1) $A = (-7, 4)$ $B = (3, -1)$ $C = (6, 1)$ $D = (K, -15)$

a) gradient of $AB = \dfrac{y_1 - y_2}{x_1 - x_2}$

$= \dfrac{4 - -1}{-7 - 3} = \dfrac{5}{-10} = -\frac{1}{2}$

b) Equation of $AB = y - y_1 = m(x - x_1)$ $m = -\frac{1}{2}$

$(x_1, y_1) = (-7, 4)$

$y - 4 = -\frac{1}{2}(x - -7)$

$2y - 8 = x - 7$

$2y + x - 1 = 0$

c) length $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-7, -3)^2 + (4 - -1)^2}$

$= \sqrt{100 + 25}$

$= \sqrt{125}$

d) co-ordinates of E =

midpoint of $AB = \dfrac{-7 + +3}{2} = \dfrac{-4}{2} = -2$

$= \dfrac{4 + -1}{2} = \dfrac{3}{2} = +1\frac{1}{2}$

co-ordinates $E = (-2, 1\frac{1}{2})$

Part (e)? use facts about perpendicular lines

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2) $\sqrt{75} - \frac{9}{\sqrt{3}} + (\sqrt{6} \times \sqrt{2})$

$\sqrt{75} = \sqrt{25 \times 3}$
$= \sqrt{25} \times \sqrt{3}$
$= 5\sqrt{3}$

$\frac{9}{\sqrt{3}} = \frac{\times\sqrt{3}}{\times\sqrt{3}} \quad \frac{9\sqrt{3}}{\cancel{3}} = 3\sqrt{3}$

$\sqrt{6} \times \sqrt{2} = \sqrt{6 \times 2}$
$= \sqrt{12}$
$= \sqrt{4}\sqrt{3}$
$= 2\sqrt{3}$

$= 5\sqrt{3} - 3\sqrt{3} + 2\sqrt{3}$
$= 4\sqrt{3}$

b) $\frac{5\sqrt{5} - 2}{4 + \sqrt{5}} \times \frac{4 - \sqrt{5}}{4 - \sqrt{5}}$

TOP

$20\sqrt{5} - 8 - 5\sqrt{5}^2 + 2\sqrt{5}$
$-5\sqrt{5} + 22\sqrt{5} - 8$
$22\sqrt{5} - 33$

7 marks out of 8

Bottom

$4^2 + \cancel{4\sqrt{5}} - \cancel{4\sqrt{5}} - \sqrt{5}^2$
$4^2 - 5 = 16 - 5$
$= 11$

Answer $= \frac{22\sqrt{5} - 33}{11} = \frac{\cancel{11}(2\sqrt{5} - 3)}{\cancel{11}}$

You MUST simplify if possible

$= 2\sqrt{5} - 3$

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4a) $y = 5x^2 + 3x - 4$ first principles

$f(x) = 5x^2 + 3x - 4$

$f(x+dx) = 5(x+dx)^2 + 3(x+dx) - 4$

$(x+dx)^2 = x^2 + 2xdx + dx^2$

$5(x+dx)^2 = 5x^2 + 10xdx + 5dx^2$

$f(x+dx) = 5x^2 + 10xdx + 5dx^2 + 3(x+dx) - 4$

$f(x+dx) = 5x^2 + 10xdx + 5dx^2 + 3x + 3dx - 4$

$$\frac{dy}{dx} = \lim_{dx \to 0} \left(\frac{f(x+dx) - f(x)}{dx} \right)$$

Consider the gradient of the chord.

$$= \frac{(\cancel{5x^2} + 10xdx + 5dx^2 + \cancel{3x} + 3dx \cancel{-4}) - (\cancel{5x^2} \cancel{+3x} \cancel{-4})}{dx}$$

$$= \frac{10xdx + 5dx^2 + 3dx}{dx}$$

$$= \frac{\cancel{dx}(10x + 5dx + 3)}{\cancel{dx}}$$

$= 10x + 5dx + 3$

now take limit as $dx \to 0$

$$\frac{dy}{dx} = 10x + 3$$

Nice

5 marks out of 5

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4b) $y = 8/x + 3\sqrt{x}$, find $\frac{dy}{dx}$ when $x = 4$.

We can not start to differentiate until there are POWERS to differentiate!

1) Write down expression
2) Re-write using powers
3) Differentiate by bringing down the power and decrease the power by 1.

$$y = 8/x + 3\sqrt{x}$$

$$y = 8x^{-1} + 3x^{1/2}$$

$$\frac{dy}{dx} = (-1)\,8x^{-1-1} + (1/2)\,3x^{\frac{1}{2}-1}$$

$$\frac{dy}{dx} = -8x^{-2} + 3/2\, x^{-1/2}$$

* We must use laws of indicies to change the powers back

$$\frac{dy}{dx} = -8/x^2 + 3/2\sqrt{x}$$

$$\frac{dy}{dx} = -8/(4)^2 + 3/2\sqrt{4}$$

$$\frac{dy}{dx} = -8/16 + 3/2(2)$$

$$\frac{dy}{dx} = -1/2 + 3/4 = 1/4$$

CONCLUDE:-
The value of the Gradient of the tangent when $x = 4$ is $1/4$

!

Very nice.
(4 marks out of 4)

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5a) $x^2 + 6x - 4$

$(x+3)^2 - 4 - (3)^2$

$(x+3)^2 - 4 - 9$

$(x+3)^2 - 13$

$a = 3$ $b = -13$

2 marks

5b) $2x^2 + 12x - 8 \equiv 2(x^2 + 6x - 4)$

Now use part (a)!

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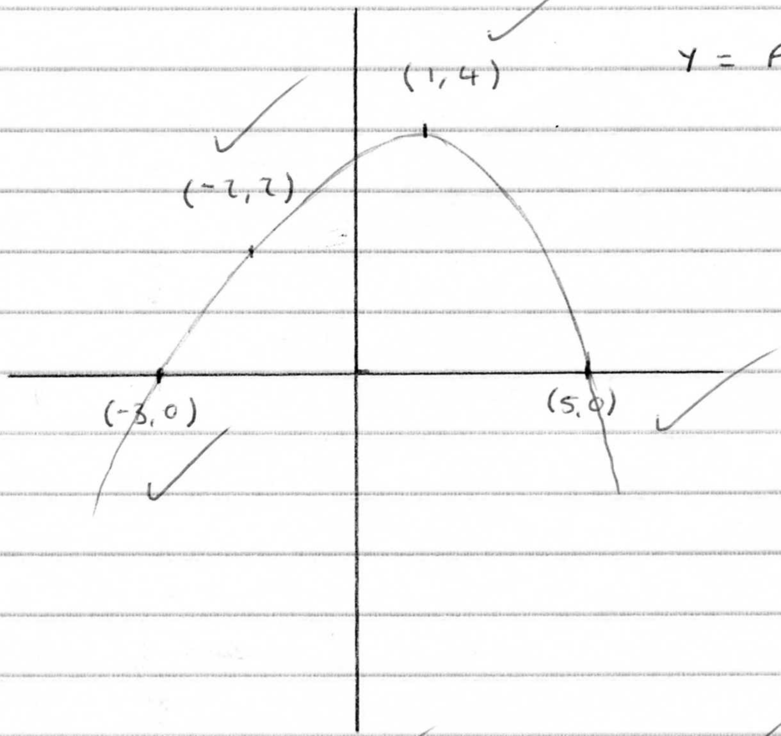
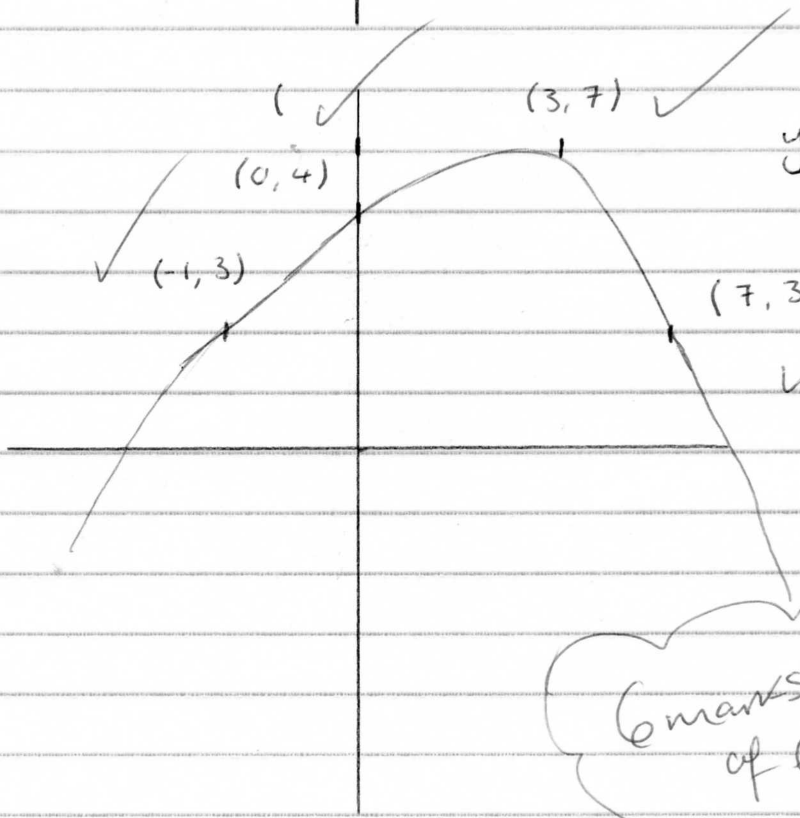
8a)



$y = f(x+2)$

Take 2 away from x co-ordinates. (Go left 2)

8b)



$y = f(x) + 3$

Add 3 on to y co-ordinates (Go up 3)

6 marks out of 6

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10a) $2x^2 - 3x - 9 \geq 0$

$2x^2 - 3x - 9 = 0$ to find the Critical Values

$(2x + 3)(x - 3) = 0$

$2x + 3 = 0$ | $x - 3 = 0$

$x = \frac{-3}{2}$ | $x = 3$ are the Critical Values

The Solution of the inequality is

$x \leq -3/2$ <u>OR</u> $x \geq 3$

3 marks out of 3

10b) TARGET

No marks here!

The discriminant.

for <u>NO</u> Real roots $b^2 - 4ac < 0$