WELSH JOINT EDUCATION COMMITTEE General Certificate of Education Advanced Subsidiary/Advanced



CYD-BWYLLGOR ADDYSG CYMRU Tystysgrif Addysg Gyffredinol Uwch Gyfrannol/Uwch

976/01

MATHEMATICS C4

Pure Mathematics

P.M. THURSDAY, 14 June 2007

 $(1\frac{1}{2}$ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer all questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. (a) Express
$$\frac{x+3}{x^2(x-1)}$$
 in terms of partial fractions. [4]
(b) Find $\int \frac{x+3}{x^2(x-1)} dx$ [2]

2. Find the equation of the tangent to the curve

$$x^5 + xy^2 + y^3 = 17$$
[4]

at the point (-1, 3).

- 3. Find all values of x in the range $0^{\circ} \le x \le 360^{\circ}$ satisfying the equation $4\cos x + 2\sin x = 3$. [7]
- 4. Expand $(1+4x)^{\frac{1}{2}} \frac{1}{1+3x}$ as far as the term in x^2 . For what range of values of x is your expansion valid? [7]
- 5. The region bounded by the curve $y = \sqrt{e^{2x} + 1}$, the *x*-axis and the lines x = 0, x = 1 is rotated through four right-angles about the *x*-axis. Find the volume of the solid generated, giving your answer correct to three decimal places. [4]
- 6. The parametric equations of the curve C are x = 2t, $y = t^2$.
 - (a) Show that the normal to C at the point P with parameter p has equation

$$x + py = p^{3} + 2p.$$
 [4]

(b) The normal to C at the point P intersects the x-axis at A and the y-axis at B. Given that O is the origin and OA = 2OB, find the value of p. [4]

7. (a) Find
$$\int x^2 \ln x \, dx$$
 . [4]

(b) Use the substitution $x = 2\sin\theta$ to show that

$$\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}} \, \mathrm{d}x = \int_0^a k \sin^2 \theta \mathrm{d}\theta \, ,$$

where the values of a and k are to be determined.

Hence, or otherwise, evaluate
$$\int_{0}^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}} \, \mathrm{d}x \, . \tag{8}$$

- 8. The price $\pounds P$ of an item at time *t* years is to be modelled as a continuous variable such that the rate of increase of *P* is directly proportional to *P*.
 - (a) Write down a differential equation that is satisfied by P. [1]
 - (b) Given that the price of the item at t = 0 is £50, show that $P = 50e^{kt}$, where k is a positive constant. [5]
 - (c) After seven years the price of the item is $\pounds 65$. Find the price of the item after sixteen years.

[4]

- 9. (a) The position vectors of the points A and B, relative to a fixed origin O, are $\mathbf{i} + 3\mathbf{j} 2\mathbf{k}$ and $3\mathbf{i} + 6\mathbf{j} + \mathbf{k}$, respectively.
 - (i) Find **AB**.
 - (ii) Find the vector equation of the line *AB*.
 - (iii) The vector equation of the line *L* is $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k} + \mu(\mathbf{i} + \mathbf{j} + 4\mathbf{k})$. Given that *L* and *AB* intersect, find the position vector of the point of intersection. [9]
 - (b) Find the angle between the vectors $\mathbf{i} + 2\mathbf{j} \mathbf{k}$ and $3\mathbf{i} \mathbf{j} + 2\mathbf{k}$. [6]
- 10. Complete the following proof by contradiction to show that, if *n* is a positive integer and $3n + 2n^3$ is odd, then *n* is odd. [2]

It is given that $3n + 2n^3$ is odd. Assume that n is even so that n = 2k.