WJEC CBAC

General Certificate of Education
Advanced Subsidiary/Advanced

975/01

MATHEMATICS C3
Pure Mathematics

A.M. FRIDAY, 11 January 2008
($1\frac{1}{2}$ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use Simpson's Rule with five ordinates to find an approximate value for

$$\int_0^{0\cdot 8} e^{x^2}\,dx.$$

Show your working and give your answer correct to four decimal places. [4]

2. *(a)* Show, by counter-example, that the statement

$$\sin 3\theta \equiv 4\sin\theta - 3\sin^3\theta$$

is false. [2]

(b) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$\sec\theta = 1 - 2\tan^2\theta.$$ [6]

3. *(a)* Given that $x = t^4 + 1$, $y = e^{2t} + 5$, find $\dfrac{dy}{dx}$ in terms of t. [4]

(b) Given that $x^4 + \sin y + x^2y^3 = 9$, find $\dfrac{dy}{dx}$ in terms of x and y. [3]

4. Show that the equation

$$2\ln(70 + x) - x = 0$$

has a root α between 8 and 9.

The recurrence relation

$$x_{n+1} = 2\ln(70 + x_n)$$

with $x_0 = 8\cdot 8$ can be used to find α.

Find and record the values of x_1, x_2, x_3. Write down the value of x_3 correct to four decimal places and prove that this value is the value of α correct to four decimal places. [7]

5. Differentiate each of the following with respect to x, simplifying your answers wherever possible.

(a) $\dfrac{\ln x}{x^2}$ *(b)* $\cos^{-1} 5x$ *(c)* $\sqrt{1 + 6x^4}$

(d) $x^3\tan 2x$ [3],[2],[2],[3]

6. *(a)* (i) Sketch the graph of $y = \ln x$.

(ii) On a separate diagram, sketch the graph of $y = |\ln x|$. [4]

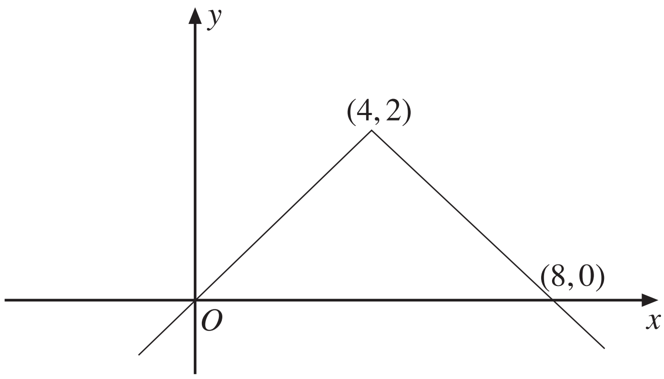
(b) Solve $|3x - 2| < 4$. [4]

7. *(a)* Find (i) $\int \sqrt{2x+3}\ \mathrm{d}x$, (ii) $\int \frac{3}{7x+2}\ \mathrm{d}x$,

(iii) $\int 5e^{2x-7}\mathrm{d}x$. [6]

(b) Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin\left(4x + \frac{\pi}{6}\right)\ \mathrm{d}x$. [4]

8. The diagram shows the graph of $y = f(x)$. The graph has its highest point at (4, 2) and it intersects the x-axis at the points (0, 0) and (8, 0).



(a) Sketch the graph of $y = 2f(x + 3)$, indicating the coordinates of the highest point and of the points where the graph intersects the x-axis. [3]

(b) On a separate diagram, sketch the graph of $y = f(2x) + 1$, indicating the coordinates of the highest point and of the point where the graph intersects the y-axis. [3]

TURN OVER

9. The functions f and g have domains $(0, \infty)$ and $(-\infty, \infty)$ respectively and are defined by

$$f(x) = \ln x,$$
$$g(x) = e^{4x}.$$

Find and simplify an expression for

(a) $fg(x)$, [2]

(b) $gf(x)$. [3]

10. The function f has domain $(2, \infty)$ and is defined by

$$f(x) = \frac{1}{\sqrt{x-2}} \quad .$$

(a) Write down the range of f. [1]

(b) Find an expression for $f^{-1}(x)$, stating the domain and range of f^{-1}. [5]

(c) Show that the equation

$$f^{-1}(x) = -\frac{3}{x}$$

has no solutions. [4]