WELSH JOINT EDUCATION COMMITTEE
General Certificate of Education
Advanced Subsidiary/Advanced

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975/01

MATHEMATICS C3

Pure Mathematics

A.M. FRIDAY, 12 January 2007

($1\frac{1}{2}$ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. *(a)* Use Simpson's Rule with five ordinates to find an approximate value for the integral

$$\int_{1}^{1\cdot8} \ln(1+x^2)\,\mathrm{d}x.$$

Show your working and give your answer correct to three decimal places. [4]

(b) Use your answer to *(a)* to deduce an approximate value for

$$\int_{1}^{1\cdot8} \ln\sqrt{1+x^2}\,\mathrm{d}x.$$

[1]

2. *(a)* Show, by counter-example, that the statement

$$\cos 3\theta \equiv 3\cos^3\theta - 4\cos\theta$$

is false. [2]

(b) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$\tan^2\theta + 2\sec\theta = 7.$$

[6]

3. Show that the equation

$$\cos x + 2x - 2 = 0$$

has a root α between 0 and $\frac{\pi}{2}$.

The recurrence relation

$$x_{n+1} = 1 - \frac{1}{2}\cos x_n$$

with $x_0 = 0\cdot5$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to three decimal places and prove that this is the value of α correct to three decimal places. [7]

4. Differentiate each of the following with respect to x and simplify your answers where possible.

(a) $(1 + 2x)^{15}$ *(b)* $\ln(1 + x^2)$ *(c)* $\dfrac{2+\cos x}{1+\sin x}$

(d) $\tan^{-1}(3x)$ *(e)* $x^2\tan x$ [2], [2], [3], [2], [2]

5. Find the x-coordinate and the nature of the stationary point of the curve given by $y = \mathrm{e}^{2x} - x - 1$. [6]

6. *(a)* Given that $x^3 + x^2y + y^4 = 17$, find $\frac{dy}{dx}$ in terms of x and y. [3]

(b) Given that $x = t^3$, $y = t^2 + 1$, find, in terms of t,

(i) $\frac{dy}{dx}$,

(ii) $\frac{d^2y}{dx^2}$. [6]

7. *(a)* Find

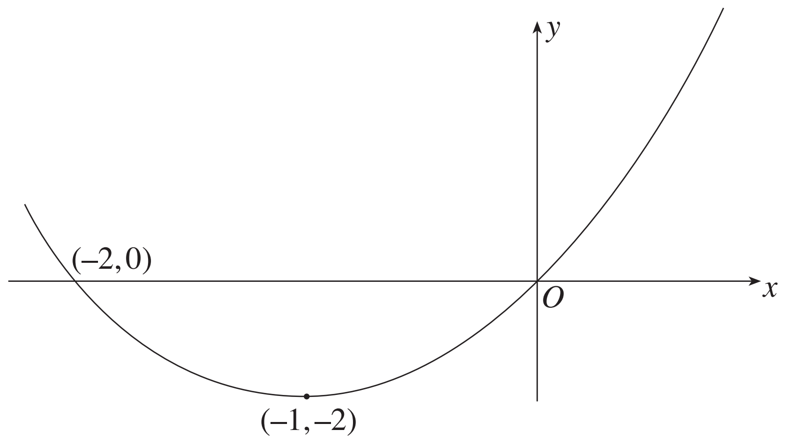
(i) $\int \frac{1}{(2x+3)^5}\ dx$

(ii) $\int e^{2-3x}\ dx$. [4]

(b) Evaluate $\int_0^2 \frac{6}{3x+2}\ dx$, expressing your answer as a single logarithm. [4]

(c) Evaluate $\int_0^{\frac{\pi}{4}} \cos\ (3x + \frac{\pi}{4})\ dx$. [4]

8. The diagram below shows a sketch of the graph of $y = f(x)$. The graph passes through the origin and the point (–2, 0), and has a minimum point at (–1, –2).



(a) Sketch the graph of $y = 2f(x - 3)$. Indicate the coordinates of the stationary point and of the points where the graph crosses the x-axis. [3]

(b) On a separate diagram, sketch the graph of $y = -f(x) + 1$. Indicate the coordinates of the stationary point and the coordinates of the point where the graph crosses the y-axis. [3]

9. The function f has domain $[1, \infty)$ and is defined by

$$f(x) = \ln(5x - 4) + 2.$$

(a) Find an expression for $f^{-1}(x)$. [4]

(b) State the domain and range of f^{-1}. [2]

10. The functions f and g are defined for all values of x by

$$f(x) = x + 5,$$

$$g(x) = |2x + 1| + 2.$$

Solve the inequality $fg(x) > 10$. [5]