C2/H   Coordinate Geometry of a Circle

① A circle C has centre D and equation

$$x^2 + y^2 + 2x - 8y + 8 = 0.$$

(a) Find the coordinates of D and the radius of C. [3]

(b) A line is drawn through the point $P\,(4, 6)$ so that it touches the circle C at the point T.

   (i) Show that $PT = \sqrt{20}$.

   (ii) Find the equation of the circle centre P which passes through the point T. [5]

② (a) Find the centre and radius of the circle C given by

$$x^2 + y^2 - 8x + 4y + 11 = 0.$$ [3]

(b) Given that the circle

$$x^2 + y^2 = a^2 \; (a > 0)$$

touches C externally, find the value of a, giving your answer correct to two decimal places. [4]

③ The circles C_1 and C_2 are given

by $\quad (x + 1)^2 + (y + 2)^2 = 25$

and $\quad x^2 + y^2 - 10x - 5y + 25 = 0$, respectively.

(a) Write down the radius and the coordinates of the centre of C_1. [2]

(b) Find the radius and the coordinates of the centre of C_2. [3]

(c) Show that C_1 and C_2 touch. [3]

④ The circle C is given by the equation

$$x^2 + y^2 - 8x + 4y - 5 = 0.$$

(a) Find the radius and the coordinates of the centre of C. [3]

(b) (i) Show that $P(1, -6)$ lies on C. [1]

   (ii) Find the equation of the tangent to C at P. [4]

(5) A circle C has equation

$$(x-5)^2 + (y-7)^2 = 25.$$

(a) Write down the radius of the circle and the coordinates of its centre. [2]

(b) Find the equation of the tangent to the circle C at the point (2,3). [4]

(c) (i) Show that $Q(13,13)$ lies outside the circle C. [2]

(ii) Find the equation of a circle with centre at Q which touches the circle C externally. [3]

(6) The circles C_1 and C_2 are given by

$$x^2 + (y+3)^2 = 1,$$
$$x^2 + y^2 - 12x - 10y - 60 = 0,$$

respectively.

(a) Write down the radius and the coordinates of the centre of C_1. [2]

(b) Find the radius and the coordinates of the centre of C_2. [3]

(c) Find the equation of the smallest circle that passes through the centres of C_1 and C_2. [5]

(7) Two circles C_1 and C_2 have equations

$$(x-10)^2 + (y-8)^2 = 4$$

and

$$x^2 + y^2 - 6x - 8y + 9 = 0.$$

(a) Write down the radius and the coordinates of the centre of C_1. [2]

(b) Find the radius and the coordinates of the centre of C_2. [3]

(c) Show that the circles do not intersect. [3]

(8) The circle C has equation

$$x^2 + y^2 - 8x - 10y + 32 = 0.$$

(a) Find the radius and the coordinates of the centre of C. [3]

(b) Show that $P(8, 8)$ lies outside C. [2]

(c) The line PT is a tangent to C touching C at T. Find the length of PT. [3]

(d) Find the equation of a circle with centre P which touches C. [3]

(9) The points (8, 4) and (2, 2) are the ends of a diameter of a circle C.

(a) Find the equation of C. [3]

(b) Find the equation of the tangent to C at the point (8, 4). [4]