973/01

MATHEMATICS C1

Pure Mathematics

A.M. MONDAY, 21 May 2007

$(1\frac{1}{2}$ hours$)$

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Calculators are **not** allowed for this paper.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The points *A, B, C, D* have coordinates (–1, 3), (1, 7), (2, –1), (5, *k*), respectively. The line *AB* is parallel to the line *CD*.

    (a)   Find the gradient of *AB*.   [2]

    (b)   Show that *k* = 5.   [3]

    (c)   The line *L* is perpendicular to *CD* and passes through the point *A*. Show that the equation of *L* is $x + 2y - 5 = 0$.   [3]

    (d)   The line *L* intersects the line *CD* at the point *E*. Find the coordinates of *E*.   [4]

2. Simplify

    (a) $2\sqrt{8} + \sqrt{18} - \dfrac{12}{\sqrt{2}}$,   [4]

    (b) $\dfrac{5 + \sqrt{15}}{5 - \sqrt{15}}$.   [4]

3. *(a)*   Given that $x - 3$ is a factor of $x^3 - 5x^2 - 2x + p$, show that $p = 24$.   [2]

    (b)   Solve the equation
    $$x^3 - 5x^2 - 2x + 24 = 0.$$ [4]

    (c)   Find the remainder when $x^3 - 5x^2 - 2x + 24$ is divided by $x - 2$.   [2]

4. *(a)*   Find the equation of the tangent to the curve $y = \dfrac{16}{x} + 3x + 2$ at the point $(4, 18)$.   [5]

    (b)   Find the solution of the simultaneous equations $y = x^2 + 2x + 4$ and $y = 4x + 7$.
            Interpret your results geometrically.   [5]

5. *(a)*   Expand $(a + b)^5$. Hence find the coefficient of x in the expansion of $\left(x + \dfrac{1}{2x}\right)^5$.   [4]

    (b)   The coefficient of x^2 in the expansion of $(1 + x)^n$ is 36. Given that n is a positive integer, find the value of n.   [3]

6. Given $y = x^2 - 12x + 10$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ from first principles.   [5]

7. *(a)* Express $2x^2 + 4x + 5$ in the form $a(x + b)^2 + c$, where a, b and c are to be determined. [3]

(b) Use the result derived in *(a)* to find the greatest value of $\dfrac{1}{2x^2 + 4x + 9}$. [2]
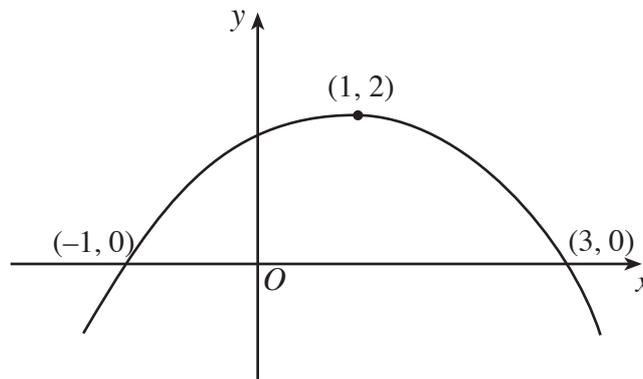
8. *(a)* Show that the equation

$$x^2 + (2k + 1)x + (k^2 + k + 1) = 0$$

has no real roots whatever the value of k. [4]

(b) Find the range of values of x satisfying the inequality

$$2x^2 + 7x + 3 < 0.$$ [3]

9.

The diagram shows the graph of $y = f(x)$. The graph passes through the points $(-1, 0)$ and $(3, 0)$ and has a maximum point at $(1, 2)$.

Sketch, on separate diagrams, the graphs of

(a) $y = f(x - 3)$, [3]

(b) $y = f\left(\dfrac{x}{2}\right)$, [3]

showing the stationary points and the points of intersection of the graphs with the x-axis.

10. The curve C has equation

$$y = x^3 - x^2 - x + 2.$$

Find the coordinates of the stationary points of C and determine the nature of each of these stationary points. [7]