C1 Jan 2009

(8) $y = 7x^2 + 5x - 2$

Let $f(x) = 7x^2 + 5x - 2$ $\quad f(x+\delta x) = 7(x+\delta x)^2 + 5(x+\delta x) - 2$

$$\frac{dy}{dx} = \lim_{\delta x \to 0}\left(\frac{f(x+\delta x) - f(x)}{\delta x}\right) \quad \text{by definition}$$

Gradient of the chord

$$\frac{f(x+\delta x)-f(x)}{\delta x} = \frac{[7(x+\delta x)^2 + 5(x+\delta x) - 2] - [7x^2 + 5x - 2]}{\delta x}$$

$$= \frac{[\cancel{7x^2} + 14x\delta x + 7\delta x^2 + \cancel{5x} + 5\delta x \cancel{-2}] - [\cancel{7x^2} + \cancel{5x} \cancel{-2}]}{\delta x}$$

$$= \frac{14x\delta x + 7\delta x^2 + 5\delta x}{\delta x}$$

$$= \frac{\cancel{\delta x}(14x + 7\delta x + 5)}{\cancel{\delta x}}$$

Now taking the limit as $\delta x \to 0$ the gradient of the chord gets closer to the gradient of the tangent

$$\frac{dy}{dx} = \lim_{\delta x \to 0}(14x + 7\delta x + 5)$$

$$\frac{dy}{dx} = 14x + 5$$

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8 b)

$$y = \frac{2}{x^3} + 5x^{2/3}$$

$$y = 2x^{-3} + 5x^{2/3}$$

$$\frac{dy}{dx} = -6x^{-4} + 5\left(\frac{2}{3}\right)x^{-1/3}$$

$$\boxed{\frac{2}{3} - 1 = \frac{-1}{3}}$$

$$\frac{dy}{dx} = \frac{-6}{x^4} + \frac{10}{3x^{1/3}}$$