GCE AS/A level

975/01

MATHEMATICS C3
PURE MATHEMATICS

P.M. WEDNESDAY, 9 June 2010

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use Simpson's Rule with five ordinates to find an approximate value for

$$\int_0^{0\cdot 8} \frac{1}{1+e^{2x}}\,dx.$$

Show your working and give your answer correct to four decimal places. [4]

2. *(a)* Show, by counter-example, that the statement

$$\cos\theta + \cos 4\theta \equiv \cos 2\theta + \cos 3\theta$$

is false. [2]

(b) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$2\tan^2\theta = \sec\theta + 8.$$ [6]

3. *(a)* Given that

$$y^4 + 4x^2y = 3x^3 - 5x,$$

find an expression for $\frac{dy}{dx}$ in terms of x and y. [4]

(b) Given that $x = 4t + \cos 2t$, $y = \sin 3t$, show that $\frac{dy}{dx} = \frac{1}{\sqrt{2}}$ when $t = \frac{\pi}{12}$. [5]

4. Show that the equation

$$4x^3 - 2x - 5 = 0$$

has a root α between 1 and 2.

The recurrence relation

$$x_{n+1} = \left(\frac{2x_n + 5}{4}\right)^{\frac{1}{3}},$$

with $x_0 = 1\cdot 2$, may be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this value is the value of α correct to five decimal places. [7]

5. *(a)* Differentiate **each** of the following with respect to x, simplifying your answer wherever possible.

(i) $(7 + 2x)^{13}$ (ii) $\sin^{-1} 5x$ (iii) $x^3 e^{4x}$ [7]

(b) By first writing $\tan x = \dfrac{\sin x}{\cos x}$, show that

$$\frac{d}{dx}(\tan x) = \sec^2 x.$$

[3]

6. *(a)* Find

(i) $\displaystyle\int \sqrt{7x - 9}\, dx$, (ii) $\displaystyle\int e^{\frac{x}{6}}\, dx$, (iii) $\displaystyle\int \frac{4}{5x - 1}\, dx$. [6]

(b) Evaluate $\displaystyle\int_2^4 \frac{8}{(3x - 4)^3}\, dx$. [4]

7. *(a)* Solve the inequality $|3x + 1| \leqslant 5$. [3]

(b) The function f is defined by $f(x) = |x|$.

(i) Sketch the graph of $y = f(x)$.

(ii) On a separate set of axes, sketch the graph of $y = f(x - 3) + 2$. On your sketch, indicate the coordinates of the point on the graph where the value of the y-coordinate is least and the coordinates of the point where the graph crosses the y-axis. [4]

8. The function g is defined by $g(x) = 3\ln(4x^2 + 9) + 2x - 7$.

(a) Show that $g'(x) = \dfrac{2(2x + 3)^2}{4x^2 + 9}$. [3]

(b) (i) Show that the graph of $y = g(x)$ has one stationary point.

(ii) Find the nature of this stationary point. [4]

TURN OVER

9. The function f has domain $[1, \infty)$ and is defined by

$$f(x) = \ln(3x - 2) + 5 .$$

(a) Find an expression for $f^{-1}(x)$. [4]

(b) State the domain of f^{-1}. [1]

10. The functions f and g have domains $[-3, \infty)$ and $(-\infty, \infty)$ respectively and are defined by

$$f(x) = \sqrt{x+4} ,$$
$$g(x) = 2x^2 - 3.$$

(a) Write down the range of f and the range of g. [2]

(b) Find an expression for $gf(x)$. Simplify your answer. [2]

(c) Solve the equation $fg(x) = 17$. [4]