# GCE AS/A level

975/01

# MATHEMATICS C3
## PURE MATHEMATICS

A.M. MONDAY, 1 June 2009

1½ hours

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

**INSTRUCTIONS TO CANDIDATES**

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

**1.** Use Simpson's Rule with five ordinates to find an approximate value for

$$\int_{1}^{1\cdot8} \sqrt{8+x^3}\ \mathrm{d}x\ .$$

Show your working and give your answer correct to four decimal places. [4]

**2.** *(a)* Show, by counter-example, that the statement

$$\cos\theta + \cos3\theta \equiv 2\cos2\theta\cos4\theta$$

is false. [2]

*(b)* Find all values of $\theta$ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$\cot^2\theta - 9 = \operatorname{cosec}\theta - \operatorname{cosec}^2\theta.$$ [6]

**3.** *(a)* Given that

$$x^3 + y^2 + x\tan 2y = 8,$$

find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $x$ and $y$. [4]

*(b)* Given that $x = 3t + t^2$, $y = \dfrac{1+4t}{3+2t}$, find

(i) $\dfrac{\mathrm{d}y}{\mathrm{d}t}$ ,

(ii) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ , simplifying your answer as much as possible. [5]

**4.** *(a)* Show that $f(x) = (2x-3)\mathrm{e}^{2x} - 4x + 5$ has a stationary value when $x$ satisfies

$$(x-1)\mathrm{e}^{2x} - 1 = 0.$$ [6]

*(b)* Show that the equation

$$(x-1)\mathrm{e}^{2x} - 1 = 0$$

has a root $\alpha$ between 1 and 2.

The recurrence relation

$$x_{n+1} = 1 + \mathrm{e}^{-2x_n}$$

with $x_0 = 1\cdot1$ may be used to find $\alpha$. Find and record the values of $x_1, x_2, x_3$. Write down the value of $x_3$ correct to four decimal places and prove that this value is the value of $\alpha$ correct to four decimal places. [7]

**5.** Differentiate each of the following with respect to $x$, simplifying your answers where possible.

*(a)* $\ln(3 + 2x^2)$ *(b)* $x^2\tan^{-1}x$ *(c)* $(5 + 7x^2)^{10}$ [2], [2], [3]

**6.** Solve the following.

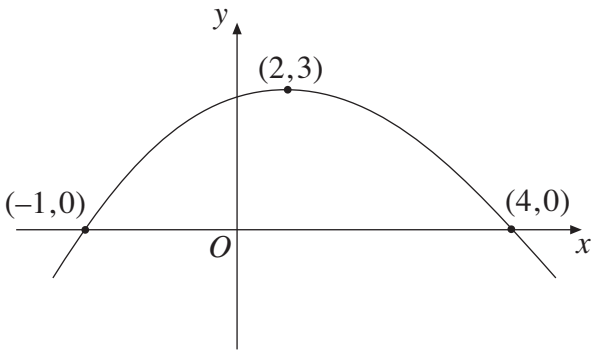
*(a)* $|9x - 7| \leqslant 3$ [3]

*(b)* $\sqrt{5|x| + 1} = 3$ [2]

**7.** *(a)* Find (i) $\int \sin 5x \, dx$, (ii) $\int \frac{3}{(2x+7)^3} \, dx$. [4]

*(b)* Evaluate $\int_0^3 \frac{2}{5x+3} \, dx$, giving your answer correct to three decimal places. [4]

**8.**




The diagram shows a sketch of the graph of $y = f(x)$. The graph has its highest point at (2, 3) and intersects the $x$-axis at the points (–1, 0) and (4, 0). Sketch the graph of $y = 3f(x - 2)$, indicating the coordinates of three points on the graph. [3]

# TURN OVER

**9.** The function $f$ has domain $(-\infty, \infty)$ and is defined by

$$f(x) = 3e^{2x} .$$

The function $g$ has domain $(0, \infty)$ and is defined by

$$g(x) = \ln 4x.$$

*(a)* Write down the domain and range of $fg$. [2]

*(b)* Solve the equation

$$fg(x) = 12.$$ [5]

**10.** The function $f$ has domain $(0, \infty)$ and is defined by

$$f(x) = 1 - \frac{2}{3x^2 + 2} .$$

*(a)* Show that $f'(x)$ is always positive. [4]

*(b)* Write down the range of $f$. [1]

*(c)* Find an expression for $f^{-1}(x)$. State the domain and range of $f^{-1}$. [6]